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Erratum on "Some Symmetry Classifications of Hyperbolic Vector Evolution Equations": *J Nonlinear Math. Phys.* **12** *suppl. 1 (2005), 13–31.*

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In section 4.2 of Ref. [1], $O(N)$-invariant vector hyperbolic equations are derived from a frame bundle formulation of geometric flows of curves $u(t,x)$ in a $N+1$-dimensional Riemannian target space M, using a parallel frame adapted to the curve. The frame equations for the curve flow are given by vanishing torsion and zero curvature of an underlying Cartan connection 1-form which as shown in [4] has a natural identification with the Maurer-Cartan form of a Lie group G. This structure implies that the space M is a flat reductive Klein geometry $G/SO(N+1)$, where G is taken to be $SO(N+2)$ or $SU(N+1)$ (under a certain dimensional reduction), but M is not flat as a homogeneous space as mistakenly claimed in [1]. More specifically, the (flat) homogeneous spaces discussed in section 4.2 should actually refer to the tangent space of M at a point x, $TM_x \simeq \mathfrak{g}/\mathfrak{s}o(N+1)$, for which there is a natural linear homogeneous structure given by the Lie algebra quotient where $\mathfrak{g}$ is $\mathfrak{s}o(N+2)$ or (a reduction of) $\mathfrak{s}u(N+1)$.

In fact these flat Klein geometries are isometric to classical Riemannian homogeneous spaces of real, symmetric type [3], with nonzero Riemannian curvature. Accordingly, the correct interpretation of the equation of the curve flow in such spaces M is a gauged sigma model [2] given by ${}^{\mathfrak{h}}\nabla_x \partial_t u^A = 0$, where u^A $(A = 1, \ldots, N+1)$ is a map into $G/SO(N+1) \simeq M$ and ${}^{\mathfrak{h}}\nabla$ is the (pullback of a) covariant derivative (differing from the ordinary derivative ∂ by nonzero Christoffel connection terms) determined by the embedding of $SO(N+1)$ in G. This covariant derivative enters the relation between the coframe e_A on M and the $SO(N+1)-$connection ${}^{\mathfrak{h}}\omega_A$ on M via ${}^{\mathfrak{h}}\nabla e_A = -{}^{\mathfrak{h}}\omega e_A$, whose curvature comes from identifying the Cartan connection in the flat Klein geometry with the zero-curvature $\mathfrak{g}$-valued 1-form ${}^{\mathfrak{g}}\omega_A = {}^{\mathfrak{h}}\omega_A + e_A$ on M.

These remarks do not affect any of the conclusions or other results in [1]. A revised version of this paper is available in electronic form: `arXiv:nlin.SI/0412015`.

References

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