Positions in the Game of Go as Complex Systems

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Outline

Rules of Go

Typical Computer Approaches

Questions

Four different spatial Scales

A new Approach

A local dynamical Systems Model

Tools for Tests

Findings

Acknowledgment
The normal board size is $19 \times 19$ but $9 \times 9$ and $13 \times 13$ are common too.
Making Moves

Two players $\bullet$ and $\bigcirc$ alternate in putting a stone on an empty intersection of the grid (called points). $\bullet$ starts.
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Making Moves

The number of stones available to both players is unlimited, in practice each side has $19^2/2 = 180$ stones. Players may pass.
Adjacent stones of one colour form what we call a chain (also called block). This diagram shows two black and one white chain.
Chains and Liberties

The extra black stone links the two black chains into a single chain. Points adjacent to a chain are called *liberties* of that chain. The black chain has 7 liberties: e3, d3, c4, d5, e6, f6, g5.
The Capture Rule

1. Rule: Each chain needs at least one liberty, for example, the white chain has one liberty. If ♦ is going to occupy that too then ..
The Capture Rule

... the white stones are captured, i.e. they are taken off the board and kept as prisoners till the end of the game. The resulting free space surrounded by stones of one colour is call eye.
2. Rule: A move that takes the last liberty of a chain of same colour without capturing an opposite chain is forbidden. That means that \(\bigcirc\) on g7 is forbidden.
.. whereas \( \times \) is allowed because it has liberties as a result of capture.
A new Quality: Life

⊗ creates two eyes. A ● move inside any one eye would violate the suicide rule because no capture would take place because ○ would still have one liberty in the other eye. Therefore, ○ cannot be captured, i.e. it is alive.
The Ko Rule

3. Rule: A move may not restore the exact position before the previous move.

The point marked K is a forbidden point for ○ to play right now. It may be played in a later move.
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- Knowledge based programs
Typical Computer Approaches

- Knowledge based programs
- Monte-Carlo programs
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- Knowledge based programs
- Monte-Carlo programs
- Learning from professional games or selfplay
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What is missing in Computer Go?

- Computer power?
- Programming Resources?
- Concepts?
- An understanding of the nature of Go?

Hardly used so far: Go has continuous aspects and much of the interaction on the board is local.
What is missing in Computer Go?

- Computer power?
What is missing in Computer Go?

- Computer power?  
- Programming Resources?
What is missing in Computer Go?

- Computer power?  ×
- Programming Resources?  (×)
- Concepts?
What is missing in Computer Go?

- Computer power?  ×
- Programming Resources?  (×)
- Concepts?  √
- An understanding of the nature of Go?
What is missing in Computer Go?

- Computer power?  ×
- Programming Resources?  (√)
- Concepts?  √
- An understanding of the nature of Go?  (√)

Hardly used so far: Go has continuous aspects and much of the interaction on the board is local.
What is missing in Computer Go?

- Computer power?  
  ×
- Programming Resources?  
  (×)
- Concepts?  
  √
- An understanding of the nature of Go?  
  (√)

Hardly used so far: Go has continuous aspects and much of the interaction on the board is local.
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Different Scales

**Ultralocal Data:** The exact shape of chains is taken into account in addition to neighbourhood relations,...
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**Local Model:** (motivated by the capture rule) The elementary objects on the board are empty points and chains which are the nodes of a graph with edges that give the neighbourhood relations of these points and chains.
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Local Model: (motivated by the capture rule) The elementary objects on the board are empty points and chains which are the nodes of a graph with edges that give the neighbourhood relations of these points and chains.

Regional elementary objects: The recursive definition of life in go has the consequence that a number of neighbouring chains and points has to be considered at once and can not be dealt with one by one, even not iteratively.
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**Ultralocal Data:** The exact shape of chains is taken into account in addition to neighbourhood relations,

**Local Model:** (motivated by the capture rule) The elementary objects on the board are empty points and chains which are the nodes of a graph with edges that give the neighbourhood relations of these points and chains.

**Regional elementary objects:** The recursive definition of *life* in go has the consequence that a number of neighbouring chains and points has to be considered at once and can not be dealt with one by one, even not iteratively.

**Global Relations:** When a Ko-fight starts then already settled regions start to depend on each other and large scale global trading starts.
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Describing a Position through a Dynamical System

The elementary objects on the board (called units from now on) are taken to be all points and chains (for which no shape is recorded). Individual stones of a chain have no own identity in this model.
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Based on the capture rule of Go, units have completely local relations among each other, i.e. the state variables describing each unit can be computed explicitly from the state variables of neighbouring units and the resulting dynamical system can be solved iteratively.
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The elementary objects on the board (called units from now on) are taken to be all points and chains (for which no shape is recorded). Individual stones of a chain have no own identity in this model.

Based on the capture rule of Go, units have completely local relations among each other, i.e. the state variables describing each unit can be computed explicitly from the state variables of neighbouring units and the resulting dynamical system can be solved iteratively.

This system couples all units on the board (i.e. all (empty) points and chains) and thus a fixpoint of the dynamical system is a ‘global consequence’ of the whole board. A change of strength of one chain would influence the strength of weak neighbouring chains and so on but would stop at strong chains.
State Variables

To each point $i$ (i.e. each empty intersection) are attached 2 real floating point type numbers:

$w_i$ . . . probability to be occupied by $\bigcirc$ at end of game

$b_i$ . . . probability to be occupied by $\bullet$ at end of game

and to each chain $j$ is attached one number:

$s_j$ . . . probability for this chain to survive.

All values are in the interval $0 . . . 1$.
For explanation purposes we also introduce

$\bar{w}_i$, $\bar{b}_i$ . . . probability that at least one neighbouring point is occupied by resp. $\bigcirc$ or $\bullet$ at the end of game.
Relations

Apart from

$$b_i + w_i = 1$$  (1)

we make the assumption  $$w_i/b_i = \bar{w}_i/\bar{b}_i$$,  i.e.

$$w_i\bar{b}_i = b_i\bar{w}_i$$  (2)

which gives correct results at least in the extreme cases  $$(\bar{w}_i, \bar{b}_i) = (1, 1), (1, 0), (0, 1)$$. From (1), (2) we get

$$w_i = \frac{\bar{w}_i}{b_i} (1 - w_i) = \frac{\bar{w}_i}{b_i} - \frac{\bar{w}_i}{b_i} w_i$$

$$w_i \left(1 + \frac{\bar{w}_i}{b_i}\right) = \frac{\bar{w}_i}{b_i}$$

$$w_i = \frac{\bar{w}_i}{b_i} \left(1 + \frac{\bar{w}_i}{b_i}\right)^{-1} = \frac{\bar{w}_i}{\bar{w}_i + \bar{b}_i}$$

where  $$\bar{w}_i, \bar{b}_i$$ have to be expressed in terms of  $$b_j, w_j, s_j$$ from the neighbouring points and chains.
Example

\[ w_i = \frac{\bar{w}_i}{\bar{w}_i + \bar{b}_i} \]

All chains are initially fully alive: \( s_j = 1 \).

\[ w_1 = \frac{1}{1+1} = \frac{1}{2} = b_1 = w_3 = b_3 \text{ (by symmetry)} \]

\[ w_2 = \frac{1}{1 + b_2} \quad \rightarrow \quad \bar{b}_2 = \text{ probability of } \bullet \text{ on 1 or } \bullet \text{ on 3} \]

\[ = \frac{1}{1 + 3/4} = 1 - \text{ probability of } \circ \text{ on 1 and } \circ \text{ on 3} \]

\[ = \frac{4}{7} = 1 - \frac{1}{4} \]

\[ b_2 = \frac{3}{7} = \frac{3}{4} \]

A similar computation is done for all chains.
Full Board Example

19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

a b c d e f g h j k l m n o p q r s t
The corresponding dynamical System

All $b$, $w$ variables are initialized to 0.5 and all $s$ variables to 1. In total there are 489 variables and as many equations:

\[ b_{t19} = - w_{t19} + 1, \]
\[ b_{s19}b_{t18} - 1 \]
\[ w_{t19} = \frac{b_{s19}b_{t18} + w_{s19}w_{t18} - 2}{b_{s19}b_{t18} + w_{s19}w_{t18} - 2} \]
\[ b_{t18} = - w_{t18} + 1, \]
\[ b_{s18}b_{t17}b_{t19} - 1 \]
\[ w_{t18} = \frac{b_{s18}b_{t17}b_{t19} + w_{s18}w_{t17}w_{t19} - 2}{b_{s18}b_{t17}b_{t19} + w_{s18}w_{t17}w_{t19} - 2} \]
\[ b_{t17} = - w_{t17} + 1, \]
\[ b_{s17}b_{t16}b_{t18} - 1 \]
\[ w_{t17} = \frac{b_{s17}b_{t16}b_{t18} + w_{s17}w_{t16}w_{t18} - 2}{b_{s17}b_{t16}b_{t18} + w_{s17}w_{t16}w_{t18} - 2} \]
\[ b_{t16} = - w_{t16} + 1, \]
\[ b_{s16}b_{t15}b_{t17} - 1 \]
\[ w_{t16} = \frac{b_{s16}b_{t15}b_{t17} + w_{s16}w_{t15}w_{t17} - 2}{b_{s16}b_{t15}b_{t17} + w_{s16}w_{t15}w_{t17} - 2} \]
The corresponding dynamical System

\begin{align*}
b_{t15} &= -w_{t15} + 1, \\
&
\quad b_{s15}b_{t14}b_{t16} - 1 \\
w_{t15} &= \frac{b_{s15}b_{t14}b_{t16} + w_{s15}w_{t14}w_{t16} - 2}{b_{s15}b_{t14}b_{t16} + w_{s15}w_{t14}w_{t16} - 2}, \\
b_{t14} &= -w_{t14} + 1, \\
&
\quad b_{t13}b_{t15}s_{s14} - 1 \\
w_{t14} &= \frac{b_{t13}b_{t15}s_{s14} - s_{s14}w_{t13}w_{t15} + w_{t13}w_{t15} - 2}{b_{t13}b_{t15}s_{s14} - s_{s14}w_{t13}w_{t15} + w_{t13}w_{t15} - 2}, \\
b_{t13} &= -w_{t13} + 1, \\
&
\quad b_{s13}b_{t12}b_{t14} - 1 \\
w_{t13} &= \frac{b_{s13}b_{t12}b_{t14} + w_{s13}w_{t12}w_{t14} - 2}{b_{s13}b_{t12}b_{t14} + w_{s13}w_{t12}w_{t14} - 2}, \\
b_{t12} &= -w_{t12} + 1, \\
&
\quad b_{s12}b_{t11}b_{t13} - 1 \\
w_{t12} &= \frac{b_{s12}b_{t11}b_{t13} + w_{s12}w_{t11}w_{t13} - 2}{b_{s12}b_{t11}b_{t13} + w_{s12}w_{t11}w_{t13} - 2}, \\
b_{t11} &= -w_{t11} + 1, \\
&
\quad b_{s11}b_{t10}b_{t12} - 1 \\
w_{t11} &= \frac{b_{s11}b_{t10}b_{t12} + w_{s11}w_{t10}w_{t12} - 2}{b_{s11}b_{t10}b_{t12} + w_{s11}w_{t10}w_{t12} - 2},
\end{align*}
The corresponding dynamical System

\[ b_{t10} = -w_{t10} + 1, \]
\[ w_{t10} = \frac{b_{s10}b_{t11}b_{t9} - 1}{b_{s10}b_{t11}b_{t9} + w_{s10}w_{t11}w_{t9} - 2}, \]
\[ b_{t9} = -w_{t9} + 1, \]
\[ w_{t9} = \frac{b_{s9}b_{t10}b_{t8} - 1}{b_{s9}b_{t10}b_{t8} + w_{s9}w_{t10}w_{t8} - 2}, \]
\[ b_{t8} = -w_{t8} + 1, \]
\[ w_{t8} = \frac{b_{t7}b_{t9}s_{s7} - b_{t7}b_{t9} + 1}{b_{t7}b_{t9}s_{s7} - b_{t7}b_{t9} - s_{s7}w_{t7}w_{t9} + 2}, \]
\[ b_{t7} = -w_{t7} + 1, \]
\[ w_{t7} = \frac{b_{t6}b_{t8}s_{s7} - b_{t6}b_{t8} + 1}{b_{t6}b_{t8}s_{s7} - b_{t6}b_{t8} - s_{s7}w_{t6}w_{t8} + 2}, \]
\[ b_{t6} = -w_{t6} + 1, \]
\[ w_{t6} = \frac{b_{s6}b_{t5}b_{t7} - 1}{b_{s6}b_{t5}b_{t7} + w_{s6}w_{t5}w_{t7} - 2}. \]
The corresponding dynamical System

\[ b_{t5} = - w_{t5} + 1, \]

\[ w_{t5} = \frac{b_{s5} b_{t4} b_{t6} - 1}{b_{s5} b_{t4} b_{t6} + w_{s5} w_{t4} w_{t6} - 2}, \]

\[ b_{t4} = - w_{t4} + 1, \]

\[ w_{t4} = \frac{b_{s4} b_{t3} b_{t5} - 1}{b_{s4} b_{t3} b_{t5} + w_{s4} w_{t3} w_{t5} - 2}, \]

\[ b_{t3} = - w_{t3} + 1, \]

\[ w_{t3} = \frac{b_{t2} b_{t4} s_{r3} - 1}{b_{t2} b_{t4} s_{r3} - s_{r3} w_{t2} w_{t4} + w_{t2} w_{t4} - 2}, \]

\[ b_{t2} = - w_{t2} + 1, \]

\[ w_{t2} = \frac{b_{s2} b_{t1} b_{t3} - 1}{b_{s2} b_{t1} b_{t3} + w_{s2} w_{t1} w_{t3} - 2}, \]

\[ b_{t1} = - w_{t1} + 1, \]

\[ w_{t1} = \frac{b_{s1} b_{t2} - 1}{b_{s1} b_{t2} + w_{s1} w_{t2} - 2}, \]
The corresponding dynamical System

\[
\begin{align*}
    b_{s19} &= -w_{s19} + 1, \\
    \quad b_{r19} b_{s18} b_{t19} &= -1, \\
    w_{s19} &= \frac{b_{r19} b_{s18} b_{t19} - 1}{b_{r19} b_{s18} b_{t19} + w_{r19} w_{s18} w_{t19} - 2}, \\
    b_{s18} &= -w_{s18} + 1, \\
    \quad b_{r18} b_{s17} b_{s19} b_{t18} &= -1, \\
    w_{s18} &= \frac{b_{r18} b_{s17} b_{s19} b_{t18} - 1}{b_{r18} b_{s17} b_{s19} b_{t18} + w_{r18} w_{s17} w_{s19} w_{t18} - 2}, \\
    b_{s17} &= -w_{s17} + 1, \\
    \quad b_{r17} b_{s16} b_{s18} b_{t17} &= -1, \\
    w_{s17} &= \frac{b_{r17} b_{s16} b_{s18} b_{t17} - 1}{b_{r17} b_{s16} b_{s18} b_{t17} + w_{r17} w_{s16} w_{s18} w_{t17} - 2}, \\
    b_{s16} &= -w_{s16} + 1, \\
    w_{s16} &= \frac{b_{s15} b_{s17} b_{t16} s_{r16} - 1}{b_{s15} b_{s17} b_{t16} s_{r16} - s_{r16} w_{s15} w_{s17} w_{t16} + w_{s15} w_{s17} w_{t16} - 2}, \\
    b_{s15} &= -w_{s15} + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
w_{s15} &= \frac{(b_{s16} b_{t15} s_{r16} s_{s14} - 1)}{(b_{s16} b_{t15} s_{r16} s_{s14} + s_{r16} s_{s14} w_{s16} w_{t15} - s_{r16} w_{s16} w_{t15} - s_{s14} w_{s16} w_{t15} + w_{s16} w_{t15} - 2)}, \\
s_{s14} &= - s_{q13} w_{s13} w_{s15} w_{t14} + 1, \\
b_{s13} &= - w_{s13} + 1, \\
w_{s13} &= \frac{(b_{s12} b_{t13} s_{r13} s_{s14} - 1)}{(b_{s12} b_{t13} s_{r13} s_{s14} + s_{r13} s_{s14} w_{s12} w_{t13} - s_{r13} w_{s12} w_{t13} - s_{s14} w_{s12} w_{t13} + w_{s12} w_{t13} - 2)}, \\
b_{s12} &= - w_{s12} + 1, \\
w_{s12} &= \frac{b_{s11} b_{s13} b_{t12} s_{r13} - 1}{b_{s11} b_{s13} b_{t12} s_{r13} - s_{r13} w_{s11} w_{s13} w_{t12} + w_{s11} w_{s13} w_{t12} - 2}, \\
b_{s11} &= - w_{s11} + 1, \\
w_{s11} &= \frac{b_{r11} b_{s10} b_{s12} b_{t11} - 1}{b_{r11} b_{s10} b_{s12} b_{t11} + w_{r11} w_{s10} w_{s12} w_{t11} - 2}
\end{align*}
\]
The corresponding dynamical System

\begin{align*}
    b_{s10} &= -w_{s10} + 1, \\
    w_{s10} &= \frac{b_{r10}b_{s11}b_{s9}b_{t10} - 1}{b_{r10}b_{s11}b_{s9}b_{t10} + w_{r10}w_{s11}w_{s9}w_{t10} - 2}, \\
    b_{s9} &= -w_{s9} + 1, \\
    w_{s9} &= \frac{b_{r9}b_{s10}b_{t9}s_{s7} - b_{r9}b_{s10}b_{t9} + 1}{b_{r9}b_{s10}b_{t9}s_{s7} - b_{r9}b_{s10}b_{t9} - s_{s7}w_{r9}w_{s10}w_{t9} + 2}, \\
    s_{s7} &= -b_{r8}b_{s6}b_{s9}b_{t7}b_{t8}s_{r7} + 1, \\
    b_{s6} &= -w_{s6} + 1, \\
    w_{s6} &= \frac{(b_{s5}b_{t6}s_{r4}s_{s7} - b_{s5}b_{t6}s_{r4} - b_{s5}b_{t6}s_{s7} + b_{s5}b_{t6} - 1)}{(b_{s5}b_{t6}s_{r4}s_{s7} - b_{s5}b_{t6}s_{r4} - b_{s5}b_{t6}s_{s7} + b_{s5}b_{t6} + s_{r4}s_{s7}w_{s5}w_{t6} - 2)}, \\
    b_{s5} &= -w_{s5} + 1, \\
    w_{s5} &= \frac{b_{s4}b_{s6}b_{t5}s_{r4} - b_{s4}b_{s6}b_{t5} + 1}{b_{s4}b_{s6}b_{t5}s_{r4} - b_{s4}b_{s6}b_{t5} - s_{r4}w_{s4}w_{s6}w_{t5} + 2}, \\
    b_{s4} &= -w_{s4} + 1,
\end{align*}
The corresponding dynamical System

\[ w_{s4} = \frac{b_{s5}b_{t4}s_{r3}s_{r4} - b_{s5}b_{t4}s_{r3} + 1}{b_{s5}b_{t4}s_{r3}s_{r4} - b_{s5}b_{t4}s_{r3} + s_{r3}s_{r4}w_{s5}w_{t4} - s_{r4}w_{s5}w_{t4} + 2}, \]

\[ b_{s2} = - w_{s2} + 1, \]

\[ b_{r2}b_{s1}b_{t2}s_{r3} - 1 \]

\[ w_{s2} = \frac{b_{r2}b_{s1}b_{t2}s_{r3} - 1}{b_{r2}b_{s1}b_{t2}s_{r3} - s_{r3}w_{r2}w_{s1}w_{t2} + w_{r2}w_{s1}w_{t2} - 2}, \]

\[ b_{s1} = - w_{s1} + 1, \]

\[ b_{r1}b_{s2}b_{t1} - 1 \]

\[ w_{s1} = \frac{b_{r1}b_{s2}b_{t1} - 1}{b_{r1}b_{s2}b_{t1} + w_{r1}w_{s2}w_{t1} - 2}, \]

\[ b_{r19} = - w_{r19} + 1, \]

\[ b_{q19}b_{r18}b_{s19} - 1 \]

\[ w_{r19} = \frac{b_{q19}b_{r18}b_{s19} - 1}{b_{q19}b_{r18}b_{s19} + w_{q19}w_{r18}w_{s19} - 2}, \]

\[ b_{r18} = - w_{r18} + 1, \]

\[ b_{q18}b_{r17}b_{r19}b_{s18} - 1 \]

\[ w_{r18} = \frac{b_{q18}b_{r17}b_{r19}b_{s18} - 1}{b_{q18}b_{r17}b_{r19}b_{s18} + w_{q18}w_{r17}w_{r19}w_{s18} - 2}, \]

\[ b_{r17} = - w_{r17} + 1, \]
The corresponding dynamical System

\[\begin{align*}
\text{w}_{r17} &= \frac{(b_{r18}b_{s17}s_{q17}s_{r16} - 1)}{(b_{r18}b_{s17}s_{q17}s_{r16} + s_{q17}s_{r16}w_{r18}w_{s17} - s_{q17}w_{r18}w_{s17} - s_{r16}w_{r18}w_{s17} + w_{r18}w_{s17} - 2)}, \\
\text{s}_{r16} &= - s_{q13}w_{q15}w_{q16}w_{r17}w_{s15}w_{s16} + 1, \\
\text{s}_{r13} &= - s_{q13}w_{p12}w_{q11}w_{r11}w_{s12}w_{s13} + 1, \\
\text{b}_{r11} &= - w_{r11} + 1, \\
\text{w}_{r11} &= \frac{b_{q11}b_{r10}b_{s11}s_{r13} - 1}{b_{q11}b_{r10}b_{s11}s_{r13} - s_{r13}w_{q11}w_{r10}w_{s11} + w_{q11}w_{r10}w_{s11} - 2}, \\
\text{b}_{r10} &= - w_{r10} + 1, \\
\text{w}_{r10} &= \frac{b_{q10}b_{r11}b_{r9}b_{s10} - 1}{b_{q10}b_{r11}b_{r9}b_{s10} + w_{q10}w_{r11}w_{r9}w_{s10} - 2}, \\
\text{b}_{r9} &= - w_{r9} + 1, \\
\text{w}_{r9} &= \frac{b_{q9}b_{r10}b_{r8}b_{s9} - 1}{b_{q9}b_{r10}b_{r8}b_{s9} + w_{q9}w_{r10}w_{r8}w_{s9} - 2},
\end{align*}\]
The corresponding dynamical System

\[
\begin{align*}
b_{r8} &= -w_{r8} + 1, \\
w_{r8} &= \frac{(b_{q8}b_{r9}s_{r7}s_{s7} - b_{q8}b_{r9}s_{r7} + 1)(b_{q8}b_{r9}s_{r7}s_{s7} - b_{q8}b_{r9}s_{r7} + s_{r7}s_{s7}w_{q8}w_{r9} - s_{s7}w_{q8}w_{r9} + 2)}{b_{q8}b_{r9}s_{r7}s_{s7} - b_{q8}b_{r9}s_{r7} + s_{s7}w_{q8}w_{r9} - s_{s7}w_{q8}w_{r9} + 2}, \\
s_{r7} &= -s_{r4}s_{s7}w_{p7}w_{q6}w_{q8}w_{r8} + 1, \\
s_{r4} &= -b_{p5}b_{q6}b_{s4}b_{s5}b_{s6}s_{q4}s_{r3}s_{r7} + 1, \\
s_{r3} &= -s_{r4}w_{q3}w_{r2}w_{s2}w_{s4}w_{t3} + 1, \\
b_{r2} &= -w_{r2} + 1, \\
w_{r2} &= \frac{b_{q2}b_{r1}b_{s2}s_{r3} + 1}{b_{q2}b_{r1}b_{s2}s_{r3}} - \frac{s_{r3}w_{q2}w_{r1}w_{s2} + w_{q2}w_{r1}w_{s2} - 2}{b_{q2}b_{r1}b_{s2}s_{r3} - w_{q2}w_{r1}w_{s2} + w_{q2}w_{r1}w_{s2} - 2}, \\
b_{r1} &= -w_{r1} + 1, \\
w_{r1} &= \frac{b_{q1}b_{r2}b_{s1} + 1}{b_{q1}b_{r2}b_{s1} + w_{q1}w_{r2}w_{s1} - 2}, \\
b_{q19} &= -w_{q19} + 1, \\
w_{q19} &= \frac{b_{p19}b_{q18}b_{r19} - 1}{b_{p19}b_{q18}b_{r19} + w_{p19}w_{q18}w_{r19} - 2},
\end{align*}
\]
The corresponding dynamical System

\[ b_{q18} = - w_{q18} + 1, \]
\[ w_{q18} = \frac{(b_{q19}b_{r18}s_{p18}s_{q17} - b_{q19}b_{r18}s_{q17} + 1)}{(b_{q19}b_{r18}s_{p18}s_{q17} - b_{q19}b_{r18}s_{q17} + s_{p18}s_{q17}w_{q19}w_{r18} - s_{p18}w_{q19}w_{r18} + 2)}, \]
\[ s_{q17} = - s_{p18}w_{q16}w_{q18}w_{r17} + 1, \]
\[ b_{q16} = - w_{q16} + 1, \]
\[ w_{q16} = \frac{(b_{q15}s_{p18}s_{q17}s_{r16} - b_{q15}s_{q17}s_{r16} + 1)}{(b_{q15}s_{p18}s_{q17}s_{r16} - b_{q15}s_{q17}s_{r16} + s_{q13}s_{r16}w_{p15}w_{q15} - s_{q13}w_{p15}w_{q15} + 2)}, \]
\[ b_{q15} = - w_{q15} + 1, \]
\[ w_{q15} = \frac{(b_{p15}b_{q16}s_{q13}s_{r16} - b_{p15}b_{q16}s_{r16} + 1)}{(b_{p15}b_{q16}s_{q13}s_{r16} - b_{p15}b_{q16}s_{r16} + s_{q13}s_{r16}w_{p15}w_{q16} - s_{q13}w_{p15}w_{q16} + 2)}, \]
\[ s_{q13} = - b_{q15}s_{p13}s_{r13}s_{r16}s_{s14} + 1, \]
\[ b_{q11} = - w_{q11} + 1, \]
The corresponding dynamical System

\[ w_{q11} \]
\[ b_{p11} \cdot b_{q10} \cdot b_{r11} \cdot s_{r13} - 1 \]
\[ = \frac{b_{p11} \cdot b_{q10} \cdot b_{r11} \cdot s_{r13} - s_{r13} \cdot w_{p11} \cdot w_{q10} \cdot w_{r11} + w_{p11} \cdot w_{q10} \cdot w_{r11}}{b_{p11} \cdot b_{q10} \cdot b_{r11} \cdot s_{r13} - s_{r13} \cdot w_{p11} \cdot w_{q10} \cdot w_{r11} + w_{p11} \cdot w_{q10} \cdot w_{r11} - 2} \]

\[ b_{q10} = - w_{q10} + 1, \]

\[ w_{q10} = \frac{b_{p10} \cdot b_{q11} \cdot b_{q9} \cdot b_{r10} - 1}{b_{p10} \cdot b_{q11} \cdot b_{q9} \cdot b_{r10} + w_{p10} \cdot w_{q11} \cdot w_{q9} \cdot w_{r10} - 2} \]

\[ b_{q9} = - w_{q9} + 1, \]

\[ w_{q9} = \frac{b_{p9} \cdot b_{q10} \cdot b_{q8} \cdot b_{r9} - 1}{b_{p9} \cdot b_{q10} \cdot b_{q8} \cdot b_{r9} + w_{p9} \cdot w_{q10} \cdot w_{q8} \cdot w_{r9} - 2} \]

\[ b_{q8} = - w_{q8} + 1, \]

\[ w_{q8} = \frac{b_{p8} \cdot b_{q9} \cdot b_{r8} \cdot s_{r7} - 1}{b_{p8} \cdot b_{q9} \cdot b_{r8} \cdot s_{r7} - s_{r7} \cdot w_{p8} \cdot w_{q9} \cdot w_{r8} + w_{p8} \cdot w_{q9} \cdot w_{r8} - 2} \]

\[ b_{q6} = - w_{q6} + 1, \]

\[ s_{p6} \cdot s_{r4} \cdot s_{r7} - s_{p6} \cdot s_{r7} - s_{r4} \cdot s_{r7} + s_{r7} - 1 \]

\[ w_{q6} = \frac{s_{p6} \cdot s_{r4} \cdot s_{r7} - s_{p6} \cdot s_{r7} - s_{r4} \cdot s_{r7} + s_{r7}}{s_{p6} \cdot s_{r4} - s_{p6} \cdot s_{r7} - s_{r4} \cdot s_{r7} + s_{r7} - 2} \]
The corresponding dynamical System

\[
\begin{align*}
  s_{q4} &= -s_{r4}w_{n4}w_{o3}w_{o5}w_{p3}w_{p5}w_{q3} + 1, \\
  b_{q3} &= -w_{q3} + 1, \\
  w_{q3} &= \frac{(b_{p3}b_{q2}s_{q4}s_{r3} - 1)}{(b_{p3}b_{q2}s_{q4}s_{r3} + s_{q4}s_{r3}w_{p3}w_{q2}} \\
  & \quad - s_{q4}w_{p3}w_{q2} - s_{r3}w_{p3}w_{q2} + w_{p3}w_{q2} - 2), \\
  b_{q2} &= -w_{q2} + 1, \\
  w_{q2} &= \frac{b_{p2}b_{q1}b_{q3}b_{r2} - 1}{b_{p2}b_{q1}b_{q3}b_{r2} + w_{p2}w_{q1}w_{q3}w_{r2} - 2}, \\
  b_{q1} &= -w_{q1} + 1, \\
  w_{q1} &= \frac{b_{p1}b_{q2}b_{r1} - 1}{b_{p1}b_{q2}b_{r1} + w_{p1}w_{q2}w_{r1} - 2}, \\
  b_{p19} &= -w_{p19} + 1, \\
  w_{p19} &= \frac{-b_{o19}b_{q19}s_{p18} - b_{o19}b_{q19} + 1}{b_{o19}b_{q19}s_{p18} - b_{o19}b_{q19} - s_{p18}w_{o19}w_{q19} + 2}, \\
  s_{p18} &= -b_{o16}b_{o17}b_{o18}b_{p15}b_{p19}b_{q16}b_{q18}s_{q17} + 1,
\end{align*}
\]
The corresponding dynamical System

\[ b_{p15} = - \ w_{p15} + 1, \]
\[ w_{p15} = \frac{(b_{q15} s_{m15} s_{p13} s_{p18} - b_{q15} s_{m15} s_{p13} - b_{q15} s_{p13} s_{p18}}{b_{q15} s_{m15} s_{p13} s_{p18} - b_{q15} s_{m15} s_{p13} - b_{q15} s_{p13} - s_{m15} s_{p13} s_{p18} w_{q15} + s_{m15} s_{p18} w_{q15} - 2}, \]
\[ s_{p13} = - s_{m15} s_{q13} w_{p12} w_{p15} + 1, \]
\[ b_{p12} = - \ w_{p12} + 1, \]
\[ w_{p12} = \frac{(b_{o12} b_{p11} s_{p13} s_{r13} - 1)}{b_{o12} b_{p11} s_{p13} s_{r13} + s_{p13} s_{r13} w_{o12} w_{p11} - s_{p13} w_{o12} w_{p11} - s_{r13} w_{o12} w_{p11} + w_{o12} w_{p11} - 2}, \]
\[ b_{p11} = - \ w_{p11} + 1, \]
\[ w_{p11} = \frac{b_{o11} b_{p10} b_{p12} b_{q11} - 1}{b_{o11} b_{p10} b_{p12} b_{q11} + w_{o11} w_{p10} w_{p12} w_{q11} - 2}, \]
\[ b_{p10} = - \ w_{p10} + 1, \]
The corresponding dynamical System

\[
\begin{align*}
\text{b}_{o10} \cdot \text{b}_{p11} \cdot \text{b}_{p9} \cdot \text{b}_{q10} - 1 \\
\text{w}_{p10} = \frac{\text{b}_{o10} \cdot \text{b}_{p11} \cdot \text{b}_{p9} \cdot \text{b}_{q10} + \text{w}_{o10} \cdot \text{w}_{p11} \cdot \text{w}_{p9} \cdot \text{w}_{q10} - 2}{\text{b}_{o10} \cdot \text{b}_{p11} \cdot \text{b}_{p9} \cdot \text{b}_{q10} + \text{w}_{o10} \cdot \text{w}_{p11} \cdot \text{w}_{p9} \cdot \text{w}_{q10} - 2} \\
\text{b}_{p9} = - \text{w}_{p9} + 1, \\
\text{w}_{p9} = \frac{\text{b}_{o9} \cdot \text{b}_{p10} \cdot \text{b}_{p8} \cdot \text{b}_{q9} - 1}{\text{b}_{o9} \cdot \text{b}_{p10} \cdot \text{b}_{p8} \cdot \text{b}_{q9} + \text{w}_{o9} \cdot \text{w}_{p10} \cdot \text{w}_{p8} \cdot \text{w}_{q9} - 2} \\
\text{b}_{p8} = - \text{w}_{p8} + 1, \\
\text{w}_{p8} = \frac{\text{b}_{o8} \cdot \text{b}_{p7} \cdot \text{b}_{p9} \cdot \text{b}_{q8} - 1}{\text{b}_{o8} \cdot \text{b}_{p7} \cdot \text{b}_{p9} \cdot \text{b}_{q8} + \text{w}_{o8} \cdot \text{w}_{p7} \cdot \text{w}_{p9} \cdot \text{w}_{q8} - 2} \\
\text{b}_{p7} = - \text{w}_{p7} + 1, \\
\text{w}_{p7} = \frac{(\text{b}_{o7} \cdot \text{b}_{p8} \cdot \text{s}_{p6} \cdot \text{s}_{r7} - \text{b}_{o7} \cdot \text{b}_{p8} \cdot \text{s}_{r7} + 1)}{(\text{b}_{o7} \cdot \text{b}_{p8} \cdot \text{s}_{p6} \cdot \text{s}_{r7} - \text{b}_{o7} \cdot \text{b}_{p8} \cdot \text{s}_{r7} + \text{s}_{p6} \cdot \text{s}_{r7} \cdot \text{w}_{o7} \cdot \text{w}_{p8} - \text{s}_{p6} \cdot \text{w}_{o7} \cdot \text{w}_{p8} + 2)} \\
\text{s}_{p6} = - \text{b}_{o6} \cdot \text{b}_{p5} \cdot \text{b}_{p7} \cdot \text{b}_{q6} + 1, \\
\text{b}_{p5} = - \text{w}_{p5} + 1,
\end{align*}
\]
The corresponding dynamical System

\[ w_{p5} = \frac{b_{o5} s_{p6} s_{q4} s_{r4} - b_{o5} s_{p6} s_{q4} - b_{o5} s_{q4} s_{r4} + b_{o5} s_{q4} - 1}{b_{o5} s_{p6} s_{q4} s_{r4} - b_{o5} s_{p6} s_{q4} - b_{o5} s_{q4} s_{r4} + b_{o5} s_{q4} - s_{p6} s_{q4} s_{r4} w_{o5} + s_{p6} s_{r4} w_{o5} - 2} , \]

\[ b_{p3} = - w_{p3} + 1 , \]

\[ w_{p3} = \frac{b_{o3} b_{p2} b_{q3} s_{q4} - 1}{b_{o3} b_{p2} b_{q3} s_{q4} - s_{q4} w_{o3} w_{p2} w_{q3} + w_{o3} w_{p2} w_{q3} - 2} , \]

\[ b_{p2} = - w_{p2} + 1 , \]

\[ w_{p2} = \frac{b_{o2} b_{p1} b_{p3} b_{q2} - 1}{b_{o2} b_{p1} b_{p3} b_{q2} + w_{o2} w_{p1} w_{p3} w_{q2} - 2} , \]

\[ b_{p1} = - w_{p1} + 1 , \]

\[ w_{p1} = \frac{b_{o1} b_{p2} b_{q1} - 1}{b_{o1} b_{p2} b_{q1} + w_{o1} w_{p2} w_{q1} - 2} , \]

\[ b_{o19} = - w_{o19} + 1 , \]

\[ w_{o19} = \frac{b_{n19} b_{o18} b_{p19} - 1}{b_{n19} b_{o18} b_{p19} + w_{n19} w_{o18} w_{p19} - 2} . \]
The corresponding dynamical System

\[
\begin{align*}
    b_{o18} &= - w_{o18} + 1, \\
    w_{o18} &= \frac{b_{n18}b_{o17}b_{o19}s_{p18} - b_{n18}b_{o17}b_{o19} + 1}{b_{n18}b_{o17}b_{o19}s_{p18} - b_{n18}b_{o17}b_{o19} + 1}, \\
    b_{o17} &= - w_{o17} + 1, \\
    w_{o17} &= \frac{b_{n16}b_{o18}s_{l16}s_{p18} - b_{n16}b_{o18}s_{l16} + 1}{b_{n16}b_{o18}s_{l16}s_{p18} - b_{n16}b_{o18}s_{l16} + s_{l16}s_{p18}w_{n16}w_{o18} - 2}, \\
    b_{o16} &= - w_{o16} + 1, \\
    w_{o16} &= \frac{b_{n16}b_{o17}s_{m15}s_{p18} - b_{n16}b_{o17}s_{m15} - b_{n16}b_{o17}s_{p18} + b_{n16}b_{o17} - 1}{b_{n16}b_{o17}s_{m15}s_{p18} - b_{n16}b_{o17}s_{m15} - b_{n16}b_{o17}s_{p18} + b_{n16}b_{o17} + s_{m15}s_{p18}w_{n16}w_{o17} - 2}, \\
    b_{o12} &= - w_{o12} + 1,
\end{align*}
\]
The corresponding dynamical System

\[ w_{o12} \]
\[
\frac{b_{n12}b_{o11}b_{p12}s_{m15} - b_{n12}b_{o11}b_{p12} + 1}{b_{n12}b_{o11}b_{p12}s_{m15} - b_{n12}b_{o11}b_{p12} + s_{m15}w_{n12}w_{o11}w_{p12} + 2} = \]

\[ b_{o11} \]
\[
- w_{o11} + 1,
\]

\[ w_{o11} \]
\[
\frac{b_{n11}b_{o10}b_{o12}b_{p11} - 1}{b_{n11}b_{o10}b_{o12}b_{p11} + w_{n11}w_{o10}w_{o12}w_{p11} - 2}
\]

\[ b_{o10} \]
\[
- w_{o10} + 1,
\]

\[ w_{o10} \]
\[
\frac{b_{n10}b_{o11}b_{o9}b_{p10} - 1}{b_{n10}b_{o11}b_{o9}b_{p10} + w_{n10}w_{o11}w_{o9}w_{p10} - 2}
\]

\[ b_{o9} \]
\[
- w_{o9} + 1,
\]

\[ w_{o9} \]
\[
\frac{b_{n9}b_{o10}b_{o8}b_{p9} - 1}{b_{n9}b_{o10}b_{o8}b_{p9} + w_{n9}w_{o10}w_{o8}w_{p9} - 2}
\]

\[ b_{o8} \]
\[
- w_{o8} + 1,
\]

\[ w_{o8} \]
\[
\frac{b_{n8}b_{o7}b_{o9}b_{p8} - 1}{b_{n8}b_{o7}b_{o9}b_{p8} + w_{n8}w_{o7}w_{o9}w_{p8} - 2}
\]
The corresponding dynamical System

\[ b_{o7} = - w_{o7} + 1, \]

\[ \frac{b_{n7} \cdot b_{o6} \cdot b_{o8} \cdot b_{p7} - 1}{b_{n7} \cdot b_{o6} \cdot b_{o8} \cdot b_{p7} + w_{n7} \cdot w_{o6} \cdot w_{o8} \cdot w_{p7} - 2} \]

\[ w_{o7} = b_{n7} \cdot b_{o6} \cdot b_{o8} \cdot b_{p7} + w_{n7} \cdot w_{o6} \cdot w_{o8} \cdot w_{p7} - 2 \]

\[ b_{o6} = - w_{o6} + 1, \]

\[ w_{o6} = \frac{(b_{o5} \cdot b_{o7} \cdot s_{n6} \cdot s_{p6} - b_{o5} \cdot b_{o7} \cdot s_{n6} - b_{o5} \cdot b_{o7} \cdot s_{p6} + b_{o5} \cdot b_{o7} - 1)}{(b_{o5} \cdot b_{o7} \cdot s_{n6} \cdot s_{p6} - b_{o5} \cdot b_{o7} \cdot s_{n6} - b_{o5} \cdot b_{o7} \cdot s_{p6} + b_{o5} \cdot b_{o7} + s_{n6} \cdot s_{p6} \cdot w_{o5} \cdot w_{o7} - 2)}, \]

\[ b_{o5} = - w_{o5} + 1, \]

\[ \frac{b_{n5} \cdot b_{o6} \cdot b_{p5} \cdot s_{q4} - 1}{b_{n5} \cdot b_{o6} \cdot b_{p5} \cdot s_{q4} - s_{q4} \cdot w_{n5} \cdot w_{o6} \cdot w_{p5} + w_{n5} \cdot w_{o6} \cdot w_{p5} - 2} \]

\[ w_{o5} = b_{n5} \cdot b_{o6} \cdot b_{p5} \cdot s_{q4} - s_{q4} \cdot w_{n5} \cdot w_{o6} \cdot w_{p5} + w_{n5} \cdot w_{o6} \cdot w_{p5} - 2 \]

\[ b_{o3} = - w_{o3} + 1, \]

\[ \frac{b_{n3} \cdot b_{o2} \cdot b_{p3} \cdot s_{q4} - 1}{b_{n3} \cdot b_{o2} \cdot b_{p3} \cdot s_{q4} - s_{q4} \cdot w_{n3} \cdot w_{o2} \cdot w_{p3} + w_{n3} \cdot w_{o2} \cdot w_{p3} - 2} \]

\[ w_{o3} = b_{n3} \cdot b_{o2} \cdot b_{p3} \cdot s_{q4} - s_{q4} \cdot w_{n3} \cdot w_{o2} \cdot w_{p3} + w_{n3} \cdot w_{o2} \cdot w_{p3} - 2 \]

\[ b_{o2} = - w_{o2} + 1, \]
The corresponding dynamical System

\[
\begin{align*}
\frac{b_n2 \cdot b_o1 \cdot b_o3 \cdot b_p2 - 1}{b_n2 \cdot b_o1 \cdot b_o3 \cdot b_p2 + w_n2 \cdot w_o1 \cdot w_o3 \cdot w_p2 - 2},
\end{align*}
\]

\[
b_o1 = -w_o1 + 1,
\]

\[
\frac{b_n1 \cdot b_o2 \cdot b_p1 - 1}{b_n1 \cdot b_o2 \cdot b_p1 + w_n1 \cdot w_o2 \cdot w_p1 - 2},
\]

\[
b_n19 = -w_n19 + 1,
\]

\[
\frac{b_m19 \cdot b_n18 \cdot b_o19 - 1}{b_m19 \cdot b_n18 \cdot b_o19 + w_m19 \cdot w_n18 \cdot w_o19 - 2},
\]

\[
b_n18 = -w_n18 + 1,
\]

\[
\frac{b_n19 \cdot b_o18 \cdot s_{l16} - 1}{b_n19 \cdot b_o18 \cdot s_{l16} - s_{l16} \cdot w_n19 \cdot w_o18 + w_n19 \cdot w_o18 - 2},
\]

\[
b_n16 = -w_n16 + 1,
\]

\[
\frac{b_o16 \cdot s_{l16} \cdot s_m15 - b_o16 \cdot s_{l16} + 1}{b_o16 \cdot s_{l16} \cdot s_m15 - b_o16 \cdot s_{l16} + s_{l16} \cdot s_m15 \cdot w_o16 - s_m15 \cdot w_o16 + 2},
\]
The corresponding dynamical System

\[ b_{n13} = -w_{n13} + 1, \]
\[ w_{n13} = \frac{b_{n12} s_{l14} s_{m14} s_{m15} - b_{n12} s_{l14} s_{m14} - b_{n12} s_{m14} s_{m15} + b_{n12} s_{m14} - 1}{b_{n12} s_{l14} s_{m14} s_{m15} - b_{n12} s_{l14} s_{m14} - b_{n12} s_{m14} s_{m15} + b_{n12} s_{m14} - s_{l14} s_{m15} w_{n12} + s_{l14} s_{m15} w_{n12} - 2}, \]
\[ b_{n12} = -w_{n12} + 1, \]
\[ w_{n12} = \frac{b_{m12} b_{n11} b_{n13} b_{o12} - 1}{b_{m12} b_{n11} b_{n13} b_{o12} + w_{m12} w_{n11} w_{n13} w_{o12} - 2}, \]
\[ b_{n11} = -w_{n11} + 1, \]
\[ w_{n11} = \frac{b_{n10} b_{n12} b_{o11} s_{m11} - b_{n10} b_{n12} b_{o11} + 1}{b_{n10} b_{n12} b_{o11} s_{m11} - b_{n10} b_{n12} b_{o11} - s_{m11} w_{n10} w_{n12} w_{o11} + 2}, \]
\[ b_{n10} = -w_{n10} + 1, \]
\[ w_{n10} = \frac{b_{m10} b_{n11} b_{n9} b_{o10} - 1}{b_{m10} b_{n11} b_{n9} b_{o10} + w_{m10} w_{n11} w_{n9} w_{o10} - 2}. \]
The corresponding dynamical System

\[ b_{n9} = - w_{n9} + 1, \]
\[ w_{n9} = \frac{b_{n10}b_{n8}b_{o9}s_{m9} - 1}{b_{n10}b_{n8}b_{o9}s_{m9} - s_{m9}w_{n10}w_{n8}w_{o9} + w_{n10}w_{n8}w_{o9} - 2}, \]
\[ b_{n8} = - w_{n8} + 1, \]
\[ w_{n8} = \frac{b_{m8}b_{n7}b_{n9}b_{o8} - 1}{b_{m8}b_{n7}b_{n9}b_{o8} + w_{m8}w_{n7}w_{n9}w_{o8} - 2}, \]
\[ b_{n7} = - w_{n7} + 1, \]
\[ w_{n7} = \frac{b_{m7}b_{n8}b_{o7}s_{n6} - b_{m7}b_{n8}b_{o7} + 1}{b_{m7}b_{n8}b_{o7}s_{n6} - b_{m7}b_{n8}b_{o7} - s_{n6}w_{m7}w_{n8}w_{o7} + 2}, \]
\[ s_{n6} = - b_{m6}b_{n5}b_{n7}b_{o6} + 1, \]
\[ b_{n5} = - w_{n5} + 1, \]
\[ w_{n5} = \frac{b_{m5}b_{n4}b_{o5}s_{n6} - b_{m5}b_{n4}b_{o5} + 1}{b_{m5}b_{n4}b_{o5}s_{n6} - b_{m5}b_{n4}b_{o5} - s_{n6}w_{m5}w_{n4}w_{o5} + 2}, \]
\[ b_{n4} = - w_{n4} + 1, \]
The corresponding dynamical System

\[ \begin{align*}
&b_{m4}b_{n3}b_{n5}s_{q4} - 1 \\
&w_{n4} = \frac{b_{m4}b_{n3}b_{n5}s_{q4} - s_{q4}w_{m4}w_{n3}w_{n5} + w_{m4}w_{n3}w_{n5} - 2}{b_{m4}b_{n3}b_{n5}s_{q4} - s_{q4}w_{m4}w_{n3}w_{n5} + w_{m4}w_{n3}w_{n5} - 2}, \\
&b_{n3} = - w_{n3} + 1, \\
&b_{m3}b_{n2}b_{n4}b_{o3} - 1 \\
&w_{n3} = \frac{b_{m3}b_{n2}b_{n4}b_{o3} + w_{m3}w_{n2}w_{n4}w_{o3} - 2}{b_{m3}b_{n2}b_{n4}b_{o3} + w_{m3}w_{n2}w_{n4}w_{o3} - 2}, \\
&b_{n2} = - w_{n2} + 1, \\
&b_{m2}b_{n1}b_{n3}b_{o2} - 1 \\
&w_{n2} = \frac{b_{m2}b_{n1}b_{n3}b_{o2} + w_{m2}w_{n1}w_{n3}w_{o2} - 2}{b_{m2}b_{n1}b_{n3}b_{o2} + w_{m2}w_{n1}w_{n3}w_{o2} - 2}, \\
&b_{n1} = - w_{n1} + 1, \\
&b_{m1}b_{n2}b_{o1} - 1 \\
&w_{n1} = \frac{b_{m1}b_{n2}b_{o1} + w_{m1}w_{n2}w_{o1} - 2}{b_{m1}b_{n2}b_{o1} + w_{m1}w_{n2}w_{o1} - 2}, \\
&b_{m19} = - w_{m19} + 1, \\
&b_{l19}b_{n19}s_{l16} - 1 \\
&w_{m19} = \frac{b_{l19}b_{n19}s_{l16} - s_{l16}w_{l19}w_{n19} + w_{l19}w_{n19} - 2}{b_{l19}b_{n19}s_{l16} - s_{l16}w_{l19}w_{n19} + w_{l19}w_{n19} - 2}.
\end{align*} \]
The corresponding dynamical System

\[
\begin{align*}
\text{s}_{m15} &= - b_{n13} b_{n16} b_{o12} b_{o16} b_{p15} s_{l16} s_{m14} s_{p13} + 1, \\
\text{s}_{m14} &= - s_{l14} s_{m15} w_{n13} + 1, \\
\text{b}_{m12} &= - w_{m12} + 1, \\
\text{w}_{m12} &= (b_{l12} b_{n12} s_{l14} s_{m11} - b_{l12} b_{n12} s_{l14} - b_{l12} b_{n12} s_{m11} + b_{l12} b_{n12} - 1) / (b_{l12} b_{n12} s_{l14} s_{m11} - b_{l12} b_{n12} s_{l14} - b_{l12} b_{n12} s_{m11} + b_{l12} b_{n12} + s_{l14} s_{m11} w_{l12} w_{n12} - 2), \\
\text{s}_{m11} &= - b_{l11} b_{m10} b_{m12} b_{n11} + 1, \\
\text{b}_{m10} &= - w_{m10} + 1, \\
\text{w}_{m10} &= (b_{l10} b_{n10} s_{m11} s_{m9} - b_{l10} b_{n10} s_{m9} + 1) / (b_{l10} b_{n10} s_{m11} s_{m9} - b_{l10} b_{n10} s_{m9} + s_{m11} s_{m9} w_{l10} w_{n10} - s_{m11} w_{l10} w_{n10} + 2), \\
\text{s}_{m9} &= - w_{l9} w_{m10} w_{m8} w_{n9} + 1, \\
\text{b}_{m8} &= - w_{m8} + 1, \\
\text{w}_{m8} &= \frac{b_{l18} b_{m7} b_{n8} s_{m9} - 1}{b_{l18} b_{m7} b_{n8} s_{m9} - s_{m9} w_{l18} w_{m7} w_{n8} + w_{l18} w_{m7} w_{n8} - 2}
\end{align*}
\]
The corresponding dynamical System

\[ b_{m7} = - w_{m7} + 1, \]
\[ w_{m7} = \frac{b_{l7} b_{m6} b_{m8} b_{n7} - 1}{b_{l7} b_{m6} b_{m8} b_{n7} + w_{l7} w_{m6} w_{m8} w_{n7} - 2}, \]
\[ b_{m6} = - w_{m6} + 1, \]
\[ w_{m6} = \frac{b_{l6} b_{m5} b_{m7} b_{n6} - b_{l6} b_{m5} b_{m7} + 1}{b_{l6} b_{m5} b_{m7} b_{n6} - b_{l6} b_{m5} b_{m7} - s_{n6} w_{l6} w_{m5} w_{m7} + 2}, \]
\[ b_{m5} = - w_{m5} + 1, \]
\[ w_{m5} = \frac{b_{l5} b_{m4} b_{m6} b_{n5} - 1}{b_{l5} b_{m4} b_{m6} b_{n5} + w_{l5} w_{m4} w_{m6} w_{n5} - 2}, \]
\[ b_{m4} = - w_{m4} + 1, \]
\[ w_{m4} = \frac{b_{l4} b_{m3} b_{m5} b_{n4} - 1}{b_{l4} b_{m3} b_{m5} b_{n4} + w_{l4} w_{m3} w_{m5} w_{n4} - 2}, \]
\[ b_{m3} = - w_{m3} + 1, \]
\[ w_{m3} = \frac{b_{l3} b_{m2} b_{m4} b_{n3} - 1}{b_{l3} b_{m2} b_{m4} b_{n3} + w_{l3} w_{m2} w_{m4} w_{n3} - 2}. \]
The corresponding dynamical System

\[ 
\begin{align*}
 b_{m2} &= - w_{m2} + 1, \\
 w_{m2} &= \frac{b_{l2} b_{m1} b_{m3} b_{n2} - 1}{b_{l2} b_{m1} b_{m3} b_{n2} + w_{l2} w_{m1} w_{m3} w_{n2} - 2}, \\
 b_{m1} &= - w_{m1} + 1, \\
 w_{m1} &= \frac{b_{l1} b_{m2} b_{m1} b_{n1} - 1}{b_{l1} b_{m2} b_{m1} b_{n1} + w_{l1} w_{m2} w_{m1} w_{n1} - 2}, \\
 b_{l19} &= - w_{l19} + 1, \\
 w_{l19} &= \frac{b_{k19} b_{m19} s_{h18} - b_{k19} b_{m19} + 1}{b_{k19} b_{m19} s_{h18} - b_{k19} b_{m19} - s_{h18} w_{k19} w_{m19} + 2}, \\
 s_{l16} &= - s_{h18} s_{l14} s_{m15} w_{k15} w_{k16} w_{m19} w_{n16} w_{n18} w_{o17} + 1, \\
 s_{l14} &= - b_{h13} b_{j12} b_{k14} b_{l12} b_{m12} b_{n13} s_{j14} s_{k12} s_{l16} s_{m14} + 1, \\
 b_{l12} &= - w_{l12} + 1, \\
 w_{l12} &= (b_{l11} b_{m12} s_{k12} s_{l14} - b_{l11} b_{m12} s_{k12} + 1)/(b_{l11} b_{m12} s_{k12} s_{l14} - b_{l11} b_{m12} s_{k12} + s_{k12} s_{l14} w_{l11} w_{m12} - s_{l14} w_{l11} w_{m12} + 2), \\
 b_{l11} &= - w_{l11} + 1,
\end{align*} \]
The corresponding dynamical System

\[ w_{l_{11}} = \frac{b_{k_{11}}b_{l_{10}}b_{l_{12}}s_{m_{11}} - b_{k_{11}}b_{l_{10}}b_{l_{12}} + 1}{b_{k_{11}}b_{l_{10}}b_{l_{12}}s_{m_{11}} - b_{k_{11}}b_{l_{10}}b_{l_{12}} - s_{m_{11}}w_{k_{11}}w_{l_{10}}w_{l_{12}} + 2}, \]

\[ b_{l_{10}} = - w_{l_{10}} + 1, \]

\[ b_{l_{9}} = - w_{l_{9}} + 1, \]

\[ b_{l_{8}} = - w_{l_{8}} + 1, \]

\[ b_{l_{7}} = - w_{l_{7}} + 1, \]

\[ b_{l_{6}} = - w_{l_{6}} + 1, \]
The corresponding dynamical System

\[
\frac{b_{k6}b_{l5}b_{l7}b_{m6} - 1}{b_{k6}b_{l5}b_{l7}b_{m6} + w_{k6}w_{l5}w_{l7}w_{m6} - 2}
\]
\[b_{l5} = - w_{l5} + 1,\]

\[
\frac{b_{k5}b_{l4}b_{l6}b_{m5} - 1}{b_{k5}b_{l4}b_{l6}b_{m5} + w_{k5}w_{l4}w_{l6}w_{m5} - 2}
\]
\[b_{l4} = - w_{l4} + 1,\]

\[
\frac{b_{k4}b_{l3}b_{l5}b_{m4} - 1}{b_{k4}b_{l3}b_{l5}b_{m4} + w_{k4}w_{l3}w_{l5}w_{m4} - 2}
\]
\[b_{l3} = - w_{l3} + 1,\]

\[
\frac{b_{l2}b_{l4}b_{m3}s_{k3} - 1}{b_{l2}b_{l4}b_{m3}s_{k3} - s_{k3}w_{l2}w_{l4}w_{m3} + w_{l2}w_{l4}w_{m3} - 2}
\]
\[b_{l2} = - w_{l2} + 1,\]

\[
\frac{b_{k2}b_{l1}b_{l3}b_{m2} - 1}{b_{k2}b_{l1}b_{l3}b_{m2} + w_{k2}w_{l1}w_{l3}w_{m2} - 2}
\]
\[b_{l1} = - w_{l1} + 1,\]
The corresponding dynamical System

\[
\begin{align*}
\text{b}_k1 \cdot \text{b}_l1 \cdot \text{b}_m1 - 1 \\
w_{l1} = \frac{\text{b}_k1 \cdot \text{b}_l1 \cdot \text{b}_m1 + w_{k1} \cdot w_{l1} \cdot w_{m1} - 2}{w_{k1} \cdot w_{l1} \cdot w_{m1} - 2},
\end{align*}
\]

\[
\begin{align*}
\text{b}_k19 &= - w_{k19} + 1, \\
w_{k19} &= \frac{\text{b}_j19 \cdot \text{b}_l19 \cdot s_{h18} - \text{b}_j19 \cdot \text{b}_l19 + 1}{\text{b}_j19 \cdot \text{b}_l19 \cdot s_{h18} - \text{b}_j19 \cdot \text{b}_l19 - s_{h18} \cdot w_{j19} \cdot w_{l19} + 2},
\end{align*}
\]

\[
\begin{align*}
\text{b}_k16 &= - w_{k16} + 1, \\
w_{k16} &= \frac{(\text{b}_k15 \cdot s_{h18} \cdot s_{j16} \cdot s_{l16} - \text{b}_k15 \cdot s_{j16} \cdot s_{l16} + 1)}{(\text{b}_k15 \cdot s_{h18} \cdot s_{j16} \cdot s_{l16} - \text{b}_k15 \cdot s_{j16} \cdot s_{l16} - s_{h18} \cdot s_{j16} \cdot s_{l16} \cdot w_{k15} \cdot s_{h18} \cdot s_{j16} \cdot s_{l16} - \text{b}_k15 \cdot s_{j16} \cdot s_{l16} - s_{h18} \cdot s_{j16} \cdot s_{l16} \cdot w_{k15} - s_{h18} \cdot s_{j16} \cdot s_{l16} \cdot w_{k15} - s_{h18} \cdot w_{k15} + 2)},
\end{align*}
\]

\[
\begin{align*}
\text{b}_k15 &= - w_{k15} + 1, \\
w_{k15} &= \frac{\text{b}_j15 \cdot \text{b}_k14 \cdot \text{b}_k16 \cdot s_{l16} - 1}{\text{b}_j15 \cdot \text{b}_k14 \cdot \text{b}_k16 \cdot s_{l16} - s_{l16} \cdot w_{j15} \cdot w_{k14} \cdot w_{k16} + w_{j15} \cdot w_{k14} \cdot w_{k16} - 2},
\end{align*}
\]

\[
\begin{align*}
\text{b}_k14 &= - w_{k14} + 1, \\
w_{k14} &= \frac{\text{b}_k15 \cdot s_{j14} \cdot s_{l14} - \text{b}_k15 \cdot s_{j14} + 1}{\text{b}_k15 \cdot s_{j14} \cdot s_{l14} - \text{b}_k15 \cdot s_{j14} + s_{j14} \cdot s_{l14} \cdot w_{k15} - s_{l14} \cdot w_{k15} + 2},
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
  s_{k12} &= - s_{l14}w_{j12}w_{k11}w_{l12} + 1, \\
  b_{k11} &= - w_{k11} + 1, \\
  w_{k11} &= \frac{b_{j11}b_{k10}b_{k9}b_{l11}s_{k12} - 1}{b_{j11}b_{k10}b_{l11}s_{k12} - s_{k12}w_{j11}w_{k10}w_{l11} + w_{j11}w_{k10}w_{l11} - 2}, \\
  b_{k10} &= - w_{k10} + 1, \\
  w_{k10} &= \frac{b_{j10}b_{k11}b_{k9}b_{l10} - 1}{b_{j10}b_{k11}b_{k9}b_{l10} + w_{j10}w_{k11}w_{k9}w_{l10} - 2}, \\
  b_{k9} &= - w_{k9} + 1, \\
  w_{k9} &= \frac{b_{j9}b_{k10}b_{k8}b_{l9} - 1}{b_{j9}b_{k10}b_{k8}b_{l9} + w_{j9}w_{k10}w_{k8}w_{l9} - 2}, \\
  b_{k8} &= - w_{k8} + 1, \\
  w_{k8} &= \frac{b_{k7}b_{k9}b_{l8}s_{j8} - 1}{b_{k7}b_{k9}b_{l8}s_{j8} - s_{j8}w_{k7}w_{k9}w_{l8} + w_{k7}w_{k9}w_{l8} - 2}, \\
  b_{k7} &= - w_{k7} + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
    w_k7 &= \frac{b_k6*b_k8*b_l7*s_j7 - b_k6*b_k8*b_l7 + 1}{b_k6*b_k8*b_l7*s_j7 - b_k6*b_k8*b_l7 - s_j7*w_k6*w_k8*w_l7 + 2}, \\
    b_k6 &= - w_k6 + 1, \\
    w_k6 &= \frac{b_j6*b_k5*b_k7*b_l6 - 1}{b_j6*b_k5*b_k7*b_l6 + w_j6*w_k5*w_k7*w_l6 - 2}, \\
    b_k5 &= - w_k5 + 1, \\
    w_k5 &= \frac{b_j5*b_k4*b_k6*b_l5 - 1}{b_j5*b_k4*b_k6*b_l5 + w_j5*w_k4*w_k6*w_l5 - 2}, \\
    b_k4 &= - w_k4 + 1, \\
    w_k4 &= \frac{b_j4*b_k5*b_l4*s_k3 - 1}{b_j4*b_k5*b_l4*s_k3 - s_k3*w_j4*w_k5*w_l4 + w_j4*w_k5*w_l4 - 2}, \\
    s_k3 &= - w_j3*w_k2*w_k4*w_l3 + 1, \\
    b_k2 &= - w_k2 + 1, \\
    w_k2 &= \frac{b_j2*b_k1*b_l2*s_k3 - 1}{b_j2*b_k1*b_l2*s_k3 - s_k3*w_j2*w_k1*w_l2 + w_j2*w_k1*w_l2 - 2}, \\
    b_k1 &= - w_k1 + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
b_{j1} & \times b_{k2} \times b_{l1} - 1 \\
w_{k1} &= \frac{b_{j1} \times b_{k2} \times b_{l1} + w_{j1} \times w_{k2} \times w_{l1} - 2}{b_{j1} \times b_{k2} \times b_{l1} + w_{j1} \times w_{k2} \times w_{l1} - 2}, \\
b_{j19} &= - w_{j19} + 1, \\
b_{h19} \times b_{k19} \times s_{h18} - b_{h19} \times b_{k19} + 1 \\
w_{j19} &= \frac{b_{h19} \times b_{k19} \times s_{h18} - b_{h19} \times b_{k19} - s_{h18} \times w_{h19} \times w_{k19} + 2}{b_{h19} \times b_{k19} \times s_{h18} - b_{h19} \times b_{k19} - s_{h18} \times w_{h19} \times w_{k19} + 2}, \\
s_{j16} &= - s_{h18} \times w_{h16} \times w_{h17} \times w_{j15} \times w_{k16} + 1, \\
b_{j15} &= - w_{j15} + 1, \\
w_{j15} &= (b_{k15} \times s_{h15} \times s_{j14} \times s_{j16} - 1)/(b_{k15} \times s_{h15} \times s_{j14} \times s_{j16} - s_{h15} \times s_{j14} \times s_{j16} \times w_{k15} + s_{h15} \times s_{j14} \times w_{k15} + s_{h15} \times s_{j16} \times w_{k15} - s_{h15} \times w_{k15} + s_{j14} \times s_{j16} \times w_{k15} - s_{j14} \times w_{k15} - s_{j16} \times w_{k15} + w_{k15} - 2), \\
s_{j14} &= - s_{l14} \times w_{h14} \times w_{j15} \times w_{k14} + 1, \\
b_{j12} &= - w_{j12} + 1, \\
w_{j12} &= (b_{j11} \times s_{h12} \times s_{k12} \times s_{l14} - b_{j11} \times s_{h12} \times s_{k12} + 1)/(b_{j11} \times s_{h12} \times s_{k12} \times s_{l14} - b_{j11} \times s_{h12} \times s_{k12} - s_{h12} \times s_{k12} \times s_{l14} \times w_{j11} + s_{h12} \times s_{l14} \times w_{j11} + s_{k12} \times s_{l14} \times w_{j11} - s_{l14} \times w_{j11} + 2),
\end{align*}
\]
The corresponding dynamical System

\[ b_{j11} = -w_{j11} + 1, \]

\[ w_{j11} \]

\[ \frac{b_{j10} \cdot b_{j12} \cdot b_{k11} \cdot s_{h12} - 1}{b_{j10} \cdot b_{j12} \cdot b_{k11} \cdot s_{h12} - s_{h12} \cdot w_{j10} \cdot w_{j12} \cdot w_{k11} + w_{j10} \cdot w_{j12} \cdot w_{k11} - 2}, \]

\[ b_{j10} = -w_{j10} + 1, \]

\[ w_{j10} \]

\[ \frac{b_{j11} \cdot b_{j9} \cdot b_{k10} \cdot s_{h12} - 1}{b_{j11} \cdot b_{j9} \cdot b_{k10} \cdot s_{h12} - s_{h12} \cdot w_{j11} \cdot w_{j9} \cdot w_{k10} + w_{j11} \cdot w_{j9} \cdot w_{k10} - 2}, \]

\[ b_{j9} = -w_{j9} + 1, \]

\[ w_{j9} \]

\[ \frac{b_{h9} \cdot b_{j10} \cdot b_{k9} \cdot s_{j8} - 1}{b_{h9} \cdot b_{j10} \cdot b_{k9} \cdot s_{j8} - s_{j8} \cdot w_{h9} \cdot w_{j10} \cdot w_{k9} + w_{h9} \cdot w_{j10} \cdot w_{k9} - 2}, \]

\[ s_{j8} = -s_{j7} \cdot w_{g8} \cdot w_{h7} \cdot w_{h9} \cdot w_{j9} \cdot w_{k8} + 1, \]

\[ s_{j7} = -b_{h7} \cdot b_{j6} \cdot b_{k7} \cdot s_{j8} + 1, \]

\[ b_{j6} = -w_{j6} + 1, \]
The corresponding dynamical system

\[ w_{j6} = \frac{(b_{j5} b_{k6} s_{h5} s_{j7} - b_{j5} b_{k6} s_{h5} - b_{j5} b_{k6} s_{j7} + b_{j5} b_{k6} - 1)}{(b_{j5} b_{k6} s_{h5} s_{j7} - b_{j5} b_{k6} s_{h5} - b_{j5} b_{k6} s_{j7} + b_{j5} b_{k6} + s_{h5} s_{j7} w_{j5} w_{k6} - 2)}, \]

\[ b_{j5} = - w_{j5} + 1, \]

\[ b_{j4} b_{j6} b_{k5} s_{h5} - b_{j4} b_{j6} b_{k5} + 1 \]

\[ w_{j5} = \frac{b_{j4} b_{j6} b_{k5} s_{h5} - b_{j4} b_{j6} b_{k5} - s_{h5} w_{j4} w_{j6} w_{k5} + 2}{b_{j4} b_{j6} b_{k5} s_{h5} - b_{j4} b_{j6} b_{k5} + w_{h4} w_{j3} w_{j5} w_{k4} - 2}, \]

\[ b_{j4} = - w_{j4} + 1, \]

\[ b_{h4} b_{j3} b_{j5} b_{k4} - 1 \]

\[ w_{j4} = \frac{b_{h4} b_{j3} b_{j5} b_{k4} + w_{h4} w_{j3} w_{j5} w_{k4} - 2}{b_{h4} b_{j3} b_{j5} b_{k4} + w_{h4} w_{j3} w_{j5} w_{k4} - 2}, \]

\[ b_{j3} = - w_{j3} + 1, \]

\[ w_{j3} = \frac{(b_{j2} b_{j4} s_{h3} s_{k3} - b_{j2} b_{j4} s_{h3} s_{k3} + 1)}{(b_{j2} b_{j4} s_{h3} s_{k3} - b_{j2} b_{j4} s_{h3} s_{k3} + s_{h3} s_{k3} w_{j2} w_{j4} - s_{h3} w_{j2} w_{j4} + 2)}, \]

\[ b_{j2} = - w_{j2} + 1, \]

\[ b_{h2} b_{j1} b_{j3} b_{k2} - 1 \]

\[ w_{j2} = \frac{b_{h2} b_{j1} b_{j3} b_{k2} + w_{h2} w_{j1} w_{j3} w_{k2} - 2}{b_{h2} b_{j1} b_{j3} b_{k2} + w_{h2} w_{j1} w_{j3} w_{k2} - 2}. \]
The corresponding dynamical System

\[ b_{j1} = - w_{j1} + 1, \]
\[ w_{j1} = \frac{b_{h1} b_{j2} b_{k1} - 1}{b_{h1} b_{j2} b_{k1} + w_{h1} w_{j2} w_{k1} - 2}, \]
\[ b_{h19} = - w_{h19} + 1, \]
\[ w_{h19} = \frac{b_{g19} b_{j19} s_{h18} - b_{g19} b_{j19} + 1}{b_{g19} b_{j19} s_{h18} - b_{g19} b_{j19} - s_{h18} w_{g19} w_{j19} + 2}, \]
\[ s_{h18} = - b_{g18} b_{h17} b_{h19} b_{j19} b_{k16} b_{k19} b_{l19} s_{j16} s_{l16} + 1, \]
\[ b_{h17} = - w_{h17} + 1, \]
\[ w_{h17} = \frac{b_{h16} s_{g17} s_{h18} s_{j16} - b_{h16} s_{g17} s_{j16} - b_{h16} s_{h18} s_{j16} + b_{h16} s_{j16} - 1}{b_{h16} s_{g17} s_{h18} s_{j16} - b_{h16} s_{g17} s_{j16} - b_{h16} s_{h18} s_{j16} + b_{h16} s_{j16} - s_{g17} s_{h18} s_{j16} w_{h16} + s_{g17} s_{h18} w_{h16} - s_{g17} w_{h16} + 2}, \]
\[ b_{h16} = - w_{h16} + 1, \]
\[ w_{h16} = \frac{b_{h17} s_{g17} s_{h15} s_{j16} - b_{h17} s_{h15} s_{j16} + 1}{b_{h17} s_{g17} s_{h15} s_{j16} - b_{h17} s_{h15} s_{j16} - s_{g17} s_{h15} s_{j16} w_{h17} + s_{g17} s_{h15} w_{h17} + s_{g17} s_{j16} w_{h17} - s_{g17} w_{h17} + 2}, \]
The corresponding dynamical System

\[
\begin{align*}
\text{s}_h_{15} &= - \text{s}_g_{17} \times \text{w}_h_{14} \times \text{w}_h_{16} \times \text{w}_j_{15} + 1, \\
\text{b}_h_{14} &= - \text{w}_h_{14} + 1, \\
\text{w}_h_{14} &= (\text{b}_h_{13} \times \text{s}_h_{12} \times \text{s}_h_{15} \times \text{s}_j_{14} - 1) / (\text{b}_h_{13} \times \text{s}_h_{12} \times \text{s}_h_{15} \times \text{s}_j_{14} \\
&\quad - \text{s}_h_{12} \times \text{s}_h_{15} \times \text{s}_j_{14} \times \text{w}_h_{13} + \text{s}_h_{12} \times \text{s}_h_{15} \times \text{w}_h_{13} + \text{s}_h_{12} \times \text{s}_j_{14} \times \text{w}_h_{13} \\
&\quad - \text{s}_h_{12} \times \text{w}_h_{13} + \text{s}_h_{15} \times \text{s}_j_{14} \times \text{w}_h_{13} - \text{s}_h_{15} \times \text{w}_h_{13} - \text{s}_j_{14} \times \text{w}_h_{13} + \text{w}_h_{13} - 2), \\
\text{b}_h_{13} &= - \text{w}_h_{13} + 1, \\
\text{w}_h_{13} &= \frac{\text{b}_h_{14} \times \text{s}_h_{12} \times \text{s}_l_{14} - \text{b}_h_{14} \times \text{s}_h_{12} + 1}{\text{b}_h_{14} \times \text{s}_h_{12} \times \text{s}_l_{14} - \text{b}_h_{14} \times \text{s}_h_{12} + \text{s}_h_{12} \times \text{s}_l_{14} \times \text{w}_h_{14} - \text{s}_l_{14} \times \text{w}_h_{14} + 2}, \\
\text{s}_h_{12} &= - \text{s}_f_{10} \times \text{s}_g_{17} \times \text{s}_g_{9} \times \text{w}_e_{14} \times \text{w}_f_{13} \times \text{w}_h_{13} \times \text{w}_h_{14} \times \text{w}_h_{9} \times \text{w}_j_{10} \times \text{w}_j_{11} \times \text{w}_j_{12} + 1, \\
\text{b}_h_{9} &= - \text{w}_h_{9} + 1, \\
\text{w}_h_{9} &= (\text{b}_j_{9} \times \text{s}_g_{9} \times \text{s}_h_{12} \times \text{s}_j_{8} - \text{b}_j_{9} \times \text{s}_h_{12} \times \text{s}_j_{8} + 1) / (\text{b}_j_{9} \times \text{s}_g_{9} \times \text{s}_h_{12} \times \text{s}_j_{8} \\
&\quad - \text{b}_j_{9} \times \text{s}_h_{12} \times \text{s}_j_{8} - \text{s}_g_{9} \times \text{s}_h_{12} \times \text{s}_j_{8} \times \text{w}_j_{9} + \text{s}_g_{9} \times \text{s}_h_{12} \times \text{w}_j_{9} \\
&\quad + \text{s}_g_{9} \times \text{s}_j_{8} \times \text{w}_j_{9} - \text{s}_g_{9} \times \text{w}_j_{9} + 2),
\end{align*}
\]
The corresponding dynamical System

\[ b_{h7} = - w_{h7} + 1, \]
\[ w_{h7} = (s_{g7} s_{h5} s_{j7} s_{j8} - s_{g7} s_{h5} s_{j8} - s_{g7} s_{j7} s_{j8} + s_{g7} s_{j8} - 1) / \]
\[ (2 s_{g7} s_{h5} s_{j7} s_{j8} - s_{g7} s_{h5} s_{j7} - s_{g7} s_{h5} s_{j8} - s_{g7} s_{j7} s_{j8} + s_{g7} s_{j8} - s_{h5} s_{j7} s_{j8} + s_{h5} s_{j7} - 2), \]
\[ s_{h5} = - b_{g6} b_{h4} b_{h7} b_{j5} b_{j6} s_{f5} + 1, \]
\[ b_{h4} = - w_{h4} + 1, \]
\[ w_{h4} = (b_{g4} b_{j4} s_{h3} s_{h5} - b_{g4} b_{j4} s_{h3} - b_{g4} b_{j4} s_{h5} + b_{g4} b_{j4} - 1) / \]
\[ (b_{g4} b_{j4} s_{h3} s_{h5} - b_{g4} b_{j4} s_{h3} - b_{g4} b_{j4} s_{h5} + b_{g4} b_{j4} + s_{h3} s_{h5} w_{g4} w_{j4} - 2), \]
\[ s_{h3} = - b_{h2} b_{h4} b_{j3} s_{g3} + 1, \]
\[ b_{h2} = - w_{h2} + 1, \]
\[ w_{h2} = (b_{h1} b_{j2} s_{g3} s_{h3} - b_{h1} b_{j2} s_{g3} + 1) / (b_{h1} b_{j2} s_{g3} s_{h3} - b_{h1} b_{j2} s_{g3} + s_{g3} s_{h3} w_{h1} w_{j2} - s_{h3} w_{h1} w_{j2} + 2), \]
\[ b_{h1} = - w_{h1} + 1, \]
The corresponding dynamical System

\[
\begin{align*}
\text{w}_{h1} &= \frac{\text{b}_{g1}\text{b}_{h2}\text{b}_{j1} - 1}{\text{b}_{g1}\text{b}_{h2}\text{b}_{j1} + \text{w}_{g1}\text{w}_{h2}\text{w}_{j1} - 2}, \\
\text{b}_{g19} &= - \text{w}_{g19} + 1, \\
\text{b}_{f19}\text{b}_{g18}\text{b}_{h19} - 1 &\quad \text{w}_{g19} = \frac{\text{b}_{f19}\text{b}_{g18}\text{b}_{h19} + \text{w}_{f19}\text{w}_{g18}\text{w}_{h19} - 2}{\text{b}_{f19}\text{b}_{g18}\text{b}_{h19} + \text{w}_{f19}\text{w}_{g18}\text{w}_{h19} - 2}, \\
\text{b}_{g18} &= - \text{w}_{g18} + 1, \\
\text{w}_{g18} &= (\text{b}_{g19}\text{s}_{d17}\text{s}_{g17}\text{s}_{h18} - \text{b}_{g19}\text{s}_{d17}\text{s}_{g17} - \text{b}_{g19}\text{s}_{d17}\text{s}_{h18} + \text{b}_{g19}\text{s}_{d17} - \text{b}_{g19}\text{s}_{g17}\text{s}_{h18} + \text{b}_{g19}\text{s}_{g17} + \text{b}_{g19}\text{s}_{h18} - \text{b}_{g19} + 1)/(\text{b}_{g19}\text{s}_{d17}\text{s}_{g17}\text{s}_{h18} - \text{b}_{g19}\text{s}_{d17}\text{s}_{g17} - \text{b}_{g19}\text{s}_{d17}\text{s}_{h18} + \text{b}_{g19}\text{s}_{d17} - \text{b}_{g19}\text{s}_{g17}\text{s}_{h18} + \text{b}_{g19}\text{s}_{g17} + \text{b}_{g19}\text{s}_{h18} - \text{b}_{g19} - \text{s}_{d17}\text{s}_{g17}\text{s}_{h18}\text{w}_{g19} + 2), \\
\text{s}_{g17} &= - \text{b}_{e14}\text{b}_{f13}\text{b}_{g18}\text{b}_{h16}\text{b}_{h17}\text{s}_{b13}\text{s}_{e16}\text{s}_{h12}\text{s}_{h15} + 1, \\
\text{s}_{g9} &= - \text{b}_{f9}\text{b}_{g8}\text{b}_{h9}\text{s}_{h12} + 1, \\
\text{b}_{g8} &= - \text{w}_{g8} + 1,
\end{align*}
\]
The corresponding dynamical System

\[ w_{g8} = \frac{(b_{f8} s_{g7} s_{g9} s_{j8} - b_{f8} s_{g7} s_{j8} + 1)/(b_{f8} s_{g7} s_{g9} s_{j8} - b_{f8} s_{g7} s_{j8} - s_{g7} s_{g9} s_{j8} w_{f8} + s_{g7} s_{g9} w_{f8} + s_{g9} s_{j8} w_{f8} - s_{g9} w_{f8} + 2),}{s_{g7} = - s_{e7} w_{f6} w_{f8} w_{g6} w_{g8} w_{h7} + 1,}

\[ b_{g6} = - w_{g6} + 1, \]

\[ w_{g6} = \frac{(b_{f6} s_{f5} s_{g7} s_{h5} - b_{f6} s_{f5} s_{g7} + 1)/(b_{f6} s_{f5} s_{g7} s_{h5} - b_{f6} s_{f5} s_{g7} - s_{f5} s_{g7} s_{h5} w_{f6} + s_{f5} s_{h5} w_{f6} + s_{g7} s_{h5} w_{f6} - s_{h5} w_{f6} + 2),}{b_{g4} = - w_{g4} + 1,}

\[ w_{g4} = \frac{(b_{f4} b_{h4} s_{f5} s_{g3} - 1)/(b_{f4} b_{h4} s_{f5} s_{g3} + s_{f5} s_{g3} w_{f4} w_{h4} - s_{f5} w_{f4} w_{h4} - s_{g3} w_{f4} w_{h4} + w_{f4} w_{h4} - 2),}{w_{g3} = - s_{h3} w_{f2} w_{f3} w_{g1} w_{g4} w_{h2} + 1,}

\[ b_{g1} = - w_{g1} + 1, \]

\[ b_{f1} b_{h1} s_{g3} - \frac{b_{f1} b_{h1} s_{g3} - s_{g3} w_{f1} w_{h1} + w_{f1} w_{h1} - 2}{b_{f1} b_{h1} s_{g3} - s_{g3} w_{f1} w_{h1} + w_{f1} w_{h1} - 2}. \]
The corresponding dynamical System

\[
\begin{align*}
b_{f19} &= -w_{f19} + 1, \\
w_{f19} &= \frac{b_{g19}s_{d17} - b_{g19} + 1}{b_{g19}s_{d17} - b_{g19} - s_{d17}w_{g19} + 2}, \\
b_{f13} &= -w_{f13} + 1, \\
w_{f13} &= \frac{s_{g17}s_{h12} - s_{h12} + 1}{2s_{g17}s_{h12} - s_{g17} - s_{h12} + 2}, \\
s_{f10} &= -b_{a11}b_{b10}b_{c9}b_{d9}b_{e9}b_{f9}s_{b13}s_{h12} + 1, \\
b_{f9} &= -w_{f9} + 1, \\
w_{f9} &= \frac{(b_{e9}b_{f8}s_{f10}s_{g9} - b_{e9}b_{f8}s_{f10} - b_{e9}b_{f8}s_{g9} + b_{e9}b_{f8} - 1)}{(b_{e9}b_{f8}s_{f10}s_{g9} - b_{e9}b_{f8}s_{f10} - b_{e9}b_{f8}s_{g9} + b_{e9}b_{f8} + s_{f10}s_{g9}w_{e9}w_{f8} - 2)}, \\
b_{f8} &= -w_{f8} + 1, \\
w_{f8} &= \frac{b_{e8}b_{f9}b_{g8}s_{g7} - 1}{b_{e8}b_{f9}b_{g8}s_{g7} - s_{g7}w_{e8}w_{f9}w_{g8} + w_{e8}w_{f9}w_{g8} - 2}, \\
b_{f6} &= -w_{f6} + 1,
\end{align*}
\]
The corresponding dynamical System

\[ w_{f6} = \frac{(b_{e6} b_{g6} s_{f5} s_{g7} - 1)}{(b_{e6} b_{g6} s_{f5} s_{g7} + s_{f5} s_{g7} w_{e6} w_{g6} - s_{f5} w_{e6} w_{g6} - s_{g7} w_{e6} w_{g6} + w_{e6} w_{g6} - 2),} \]

\[ s_{f5} = - s_{e4} s_{h5} w_{f4} w_{f6} w_{g4} w_{g6} + 1, \]

\[ b_{f4} = - w_{f4} + 1, \]

\[ w_{f4} = \frac{(b_{f3} b_{g4} s_{e4} s_{f5} - b_{f3} b_{g4} s_{f5} + 1)}{(b_{f3} b_{g4} s_{e4} s_{f5} - b_{f3} b_{g4} s_{f5} + s_{e4} s_{f5} w_{f3} w_{g4} - s_{e4} w_{f3} w_{g4} + 2),} \]

\[ b_{f3} = - w_{f3} + 1, \]

\[ w_{f3} = \frac{b_{e3} b_{f2} b_{f4} s_{g3} - 1}{b_{e3} b_{f2} b_{f4} s_{g3} - s_{g3} w_{e3} w_{f2} w_{f4} + w_{e3} w_{f2} w_{f4} - 2} \]

\[ b_{f2} = - w_{f2} + 1, \]

\[ w_{f2} = \frac{(b_{f1} b_{f3} s_{e2} s_{g3} - 1)}{(b_{f1} b_{f3} s_{e2} s_{g3} + s_{e2} s_{g3} w_{f1} w_{f3} - s_{e2} w_{f1} w_{f3} - s_{g3} w_{f1} w_{f3} + w_{f1} w_{f3} - 2),} \]

\[ b_{f1} = - w_{f1} + 1, \]

\[ w_{f1} = \frac{b_{e1} b_{f2} b_{g1} - 1}{b_{e1} b_{f2} b_{g1} + w_{e1} w_{f2} w_{g1} - 2} \]
The corresponding dynamical System

\[s_{e16} = - s_{b14} s_{d17} s_{g17} w_{a16} w_{b17} w_{c18} + 1,\]
\[b_{e14} = - w_{e14} + 1,\]
\[w_{e14} = \frac{s_{g17} s_{h12} - s_{h12} + 1}{2 s_{g17} s_{h12} - s_{g17} - s_{h12} + 2},\]
\[b_{e9} = - w_{e9} + 1,\]
\[w_{e9} = \frac{b_{d9} b_{e8} b_{f9} s_{f10} - b_{d9} b_{e8} b_{f9} + 1}{b_{d9} b_{e8} b_{f9} s_{f10} - b_{d9} b_{e8} b_{f9} - s_{f10} w_{d9} w_{e8} w_{f9} + 2},\]
\[b_{e8} = - w_{e8} + 1,\]
\[w_{e8} = \frac{b_{d8} b_{e9} b_{f8} s_{e7} - b_{d8} b_{e9} b_{f8} + 1}{b_{d8} b_{e9} b_{f8} s_{e7} - b_{d8} b_{e9} b_{f8} - s_{e7} w_{d8} w_{e9} w_{f8} + 2},\]
\[s_{e7} = - b_{d6} b_{d8} b_{e6} b_{e8} s_{c7} s_{g7} + 1,\]
\[b_{e6} = - w_{e6} + 1,\]
\[w_{e6} = (b_{d6} b_{f6} s_{e4} s_{e7} - b_{d6} b_{f6} s_{e4} - b_{d6} b_{f6} s_{e7} + b_{d6} b_{f6} - 1) / (b_{d6} b_{f6} s_{e4} s_{e7} - b_{d6} b_{f6} s_{e4} - b_{d6} b_{f6} s_{e7} + b_{d6} b_{f6} + s_{e4} s_{e7} w_{d6} w_{f6} - 2),\]
The corresponding dynamical System

\[
\begin{align*}
\text{s\_e4} &= - \text{b\_d4}\text{b\_d6}\text{b\_e3}\text{b\_e6}\text{b\_f4}\text{s\_c5}\text{s\_f5} + 1, \\
\text{b\_e3} &= - \text{w\_e3} + 1, \\
\text{w\_e3} &= (\text{b\_f3}\text{s\_d3}\text{s\_e2}\text{s\_e4} - \text{b\_f3}\text{s\_d3}\text{s\_e2} - \text{b\_f3}\text{s\_e2}\text{s\_e4} + \text{b\_f3}\text{s\_e2} - 1) / \\
&\quad (\text{b\_f3}\text{s\_d3}\text{s\_e2}\text{s\_e4} - \text{b\_f3}\text{s\_d3}\text{s\_e2} - \text{b\_f3}\text{s\_e2}\text{s\_e4} + \text{b\_f3}\text{s\_e2} - \text{s\_d3}\text{s\_e2}\text{s\_e4}\text{w\_f3} + \text{s\_d3}\text{s\_e4}\text{w\_f3} - 2), \\
\text{s\_e2} &= - \text{s\_d3}\text{w\_e1}\text{w\_e3}\text{w\_f2} + 1, \\
\text{b\_e1} &= - \text{w\_e1} + 1, \\
\text{w\_e1} &= - \frac{\text{b\_f1}\text{s\_d1}\text{s\_e2} - 1}{\text{b\_f1}\text{s\_d1}\text{s\_e2} + \text{s\_d1}\text{s\_e2}\text{w\_f1} - \text{s\_d1}\text{w\_f1} - \text{s\_e2}\text{w\_f1} + \text{w\_f1} - 2}, \\
\text{b\_d19} &= - \text{w\_d19} + 1, \\
\text{w\_d19} &= - \frac{\text{s\_c19}\text{s\_d17}\text{s\_d18} - \text{s\_c19}\text{s\_d18} + 1}{\text{s\_c19}\text{s\_d17} - \text{s\_c19}\text{s\_d18} + \text{s\_d17}\text{s\_d18} - \text{s\_d17} + 2}, \\
\text{s\_d18} &= - \text{s\_d17}\text{w\_c18}\text{w\_d19} + 1, \\
\text{s\_d17} &= - \text{b\_d19}\text{b\_f19}\text{b\_g18}\text{s\_d18}\text{s\_e16} + 1, \\
\text{b\_d9} &= - \text{w\_d9} + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
\frac{b_{c9}b_{d8}b_{e9}s_{f10} - b_{c9}b_{d8}b_{e9} + 1}{b_{c9}b_{d8}b_{e9}s_{f10} - b_{c9}b_{d8}b_{e9} + 1} &= w_{d9}, \\
\frac{b_{c8}b_{d9}b_{e8}s_{e7} - b_{c8}b_{d9}b_{e8} + 1}{b_{c8}b_{d9}b_{e8}s_{e7} - b_{c8}b_{d9}b_{e8} + 1} &= w_{d8}, \\
\frac{b_{d6} - w_{d6} + 1}{w_{d6} = (b_{c6}b_{e6}s_{e4}s_{e7} - b_{c6}b_{e6}s_{e4} - b_{c6}b_{e6}s_{e7} + b_{c6}b_{e6} - 1)/ (b_{c6}b_{e6}s_{e4}s_{e7} - b_{c6}b_{e6}s_{e4} - b_{c6}b_{e6}s_{e7} + b_{c6}b_{e6} + s_{e4}s_{e7}w_{c6}w_{e6} - 2),} \\
\frac{s_{d3} - b_{c3}b_{d4}b_{e3}s_{c2}s_{d1}s_{e2} + 1}{s_{d3} = - b_{c3}b_{d4}b_{e3}s_{c2}s_{d1}s_{e2} + 1}
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
    s_{d1} &= - s_{d3} w_{c1} w_{e1} + 1, \\
    s_{c19} &= - w_{b19} w_{c18} w_{d19} + 1, \\
    b_{c18} &= - w_{c18} + 1, \\
    w_{c18} &= (s_{b18} s_{c19} s_{d18} s_{e16} - 1)/(2 s_{b18} s_{c19} s_{d18} s_{e16} \\
    &\quad - s_{b18} s_{c19} s_{d18} - s_{b18} s_{c19} s_{e16} + s_{b18} s_{c19} - s_{b18} s_{d18} s_{e16} \\
    &\quad + s_{b18} s_{d18} + s_{b18} s_{e16} - s_{b18} - s_{c19} s_{d18} s_{e16} + s_{c19} s_{d18} \\
    &\quad + s_{c19} s_{e16} - s_{c19} + s_{d18} s_{e16} - s_{d18} - s_{e16} - 1), \\
    b_{c9} &= - w_{c9} + 1, \\
    w_{c9} &= \frac{b_{b9} b_{c8} b_{d9} s_{f10} - b_{b9} b_{c8} b_{d9} + 1}{b_{b9} b_{c8} b_{d9} s_{f10} - b_{b9} b_{c8} b_{d9} - s_{f10} w_{b9} w_{c8} w_{d9} + 2}, \\
    b_{c8} &= - w_{c8} + 1, \\
    w_{c8} &= \frac{b_{b8} b_{c9} b_{d8} s_{c7} - 1}{b_{b8} b_{c9} b_{d8} s_{c7} - s_{c7} w_{b8} w_{c9} w_{d8} + w_{b8} w_{c9} w_{d8} - 2}, \\
    s_{c7} &= - s_{e7} w_{b7} w_{c6} w_{c8} + 1, \\
    b_{c6} &= - w_{c6} + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
w_{c6} &= \frac{(b_{b6}b_{d6}s_{c5}s_{c7} - 1)}{(b_{b6}b_{d6}s_{c5}s_{c7} + s_{c5}s_{c7}w_{b6}w_{d6} - s_{c5}w_{b6}w_{d6} - s_{c7}w_{b6}w_{d6} + w_{b6}w_{d6} - 2)}, \\
s_{c5} &= -s_{b4}s_{e4}w_{b5}w_{c6} + 1, \\
b_{c3} &= -w_{c3} + 1, \\
w_{c3} &= \frac{(s_{b3}s_{b4}s_{c2}s_{d3} - s_{b3}s_{b4}s_{c2} - s_{b3}s_{c2}s_{d3} + s_{b3}s_{c2} - 1)}{(s_{b3}s_{b4}s_{c2}s_{d3} + s_{b3}s_{b4}s_{d3} - s_{b3}s_{c2}s_{d3} + s_{b4}s_{d3} - 2)}, \\
s_{c2} &= -s_{d3}w_{b2}w_{c1}w_{c3} + 1, \\
b_{c1} &= -w_{c1} + 1, \\
\frac{b_{b1}s_{c2}s_{d1} - 1}{b_{b1}s_{c2}s_{d1} + s_{c2}s_{d1}w_{b1} - s_{c2}w_{b1} - s_{d1}w_{b1} + w_{b1} - 2}, \\
b_{b19} &= -w_{b19} + 1, \\
w_{b19} &= \frac{(b_{a19}s_{b18}s_{c19} - 1)}{(b_{a19}s_{b18}s_{c19} + s_{b18}s_{c19}w_{a19} - s_{b18}w_{a19} - s_{c19}w_{a19} + w_{a19} - 2)}, \\
s_{b18} &= -w_{a18}w_{b17}w_{b19}w_{c18} + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
\text{b}_b17 &= - w_b17 + 1, \\
\text{w}_b17 &= \frac{(\text{b}_a17 * \text{s}_b18 * \text{s}_e16 - 1)}{(\text{b}_a17 * \text{s}_b18 * \text{s}_e16 + \text{s}_b18 * \text{s}_e16 * \text{w}_a17 - \text{s}_b18 * \text{w}_a17 - \text{s}_e16 * \text{w}_a17 + \text{w}_a17 - 2)}, \\
\text{s}_b14 &= - \text{b}_a14 * \text{s}_a15 * \text{s}_b13 * \text{s}_e16 + 1, \\
\text{s}_b13 &= - \text{s}_b14 * \text{s}_f10 * \text{s}_g17 * \text{w}_a12 * \text{w}_a13 + 1, \\
\text{b}_b10 &= - w_b10 + 1, \\
\text{w}_b10 &= \frac{\text{b}_a10 * \text{b}_b9 * \text{s}_f10 - \text{b}_a10 * \text{b}_b9 + 1}{\text{b}_a10 * \text{b}_b9 * \text{s}_f10 - \text{b}_a10 * \text{b}_b9 - \text{s}_f10 * \text{w}_a10 * \text{w}_b9 + 2}, \\
\text{b}_b9 &= - w_b9 + 1, \\
\text{w}_b9 &= \frac{\text{b}_a9 * \text{b}_b10 * \text{b}_b8 * \text{b}_c9 - 1}{\text{b}_a9 * \text{b}_b10 * \text{b}_b8 * \text{b}_c9 + \text{w}_a9 * \text{w}_b10 * \text{w}_b8 * \text{w}_c9 - 2}, \\
\text{b}_b8 &= - w_b8 + 1, \\
\text{w}_b8 &= \frac{\text{b}_a8 * \text{b}_b7 * \text{b}_b9 * \text{b}_c8 - 1}{\text{b}_a8 * \text{b}_b7 * \text{b}_b9 * \text{b}_c8 + \text{w}_a8 * \text{w}_b7 * \text{w}_b9 * \text{w}_c8 - 2}, \\
\text{b}_b7 &= - w_b7 + 1,
\end{align*}
\]
The corresponding dynamical System

\[
\begin{align*}
\text{b}_a7*\text{b}_b6*\text{b}_b8*\text{s}_c7 & - 1 \\
\text{w}_b7 = & \frac{\text{b}_a7*\text{b}_b6*\text{b}_b8*\text{s}_c7 - 1}{\text{b}_a7*\text{b}_b6*\text{b}_b8*\text{s}_c7 - \text{s}_c7*\text{w}_a7*\text{w}_b6*\text{w}_b8 + \text{w}_a7*\text{w}_b6*\text{w}_b8 - 2}, \\
\text{b}_b6 & = - \text{w}_b6 + 1, \\
\text{b}_a6*\text{b}_b5*\text{b}_b7*\text{b}_c6 & - 1 \\
\text{w}_b6 = & \frac{\text{b}_a6*\text{b}_b5*\text{b}_b7*\text{b}_c6 - 1}{\text{b}_a6*\text{b}_b5*\text{b}_b7*\text{b}_c6 + \text{w}_a6*\text{w}_b5*\text{w}_b7*\text{w}_c6 - 2}, \\
\text{b}_b5 & = - \text{w}_b5 + 1, \\
\text{w}_b5 = & \frac{(\text{b}_a5*\text{b}_b6*\text{s}_b4*\text{s}_c5 - \text{b}_a5*\text{b}_b6*\text{s}_c5 + 1)/(\text{b}_a5*\text{b}_b6*\text{s}_b4*\text{s}_c5 - \text{b}_a5*\text{b}_b6*\text{s}_c5 + \text{s}_b4*\text{s}_c5*\text{w}_a5*\text{w}_b6 - \text{s}_b4*\text{w}_a5*\text{w}_b6 + 2),}{\text{s}_b4 - \text{b}_a4*\text{b}_b5*\text{b}_c3*\text{b}_d4*\text{s}_b3*\text{s}_c5 + 1,} \\
\text{s}_b4 & = - \text{b}_a4*\text{b}_b5*\text{b}_c3*\text{b}_d4*\text{s}_b3*\text{s}_c5 + 1, \\
\text{s}_b3 & = - \text{s}_b4*\text{w}_a3*\text{w}_b2*\text{w}_c3 + 1, \\
\text{b}_b2 & = - \text{w}_b2 + 1, \\
\text{w}_b2 = & \frac{(\text{b}_a2*\text{b}_b1*\text{s}_b3*\text{s}_c2 - 1)/(\text{b}_a2*\text{b}_b1*\text{s}_b3*\text{s}_c2 + \text{s}_b3*\text{s}_c2*\text{w}_a2*\text{w}_b1 - \text{s}_b3*\text{w}_a2*\text{w}_b1 - \text{s}_c2*\text{w}_a2*\text{w}_b1 + \text{w}_a2*\text{w}_b1 - 2),}{\text{b}_b1 - \text{w}_b1 + 1,} \\
\text{b}_b1 & = - \text{w}_b1 + 1, \\
\text{b}_a1*\text{b}_b2*\text{b}_c1 & - 1 \\
\text{w}_b1 = & \frac{\text{b}_a1*\text{b}_b2*\text{b}_c1 - 1}{\text{b}_a1*\text{b}_b2*\text{b}_c1 + \text{w}_a1*\text{w}_b2*\text{w}_c1 - 2}, \\
\end{align*}
\]
The corresponding dynamical System

\[ b_{a19} = -w_{a19} + 1, \]
\[ w_{a19} = \frac{b_{a18}b_{b19} - 1}{b_{a18}b_{b19} + w_{a18}w_{b19} - 2}, \]
\[ b_{a18} = -w_{a18} + 1, \]
\[ w_{a18} = \frac{b_{a17}b_{a19}s_{b18} - 1}{b_{a17}b_{a19}s_{b18} - s_{b18}w_{a17}w_{a19} + w_{a17}w_{a19} - 2}, \]
\[ b_{a17} = -w_{a17} + 1, \]
\[ w_{a17} = \frac{b_{a16}b_{a18}b_{b17} - 1}{b_{a16}b_{a18}b_{b17} + w_{a16}w_{a18}w_{b17} - 2}, \]
\[ b_{a16} = -w_{a16} + 1, \]
\[ w_{a16} = \frac{(b_{a17}s_{a15}s_{e16} - 1)(b_{a17}s_{a15}s_{e16} + s_{a15}s_{e16}w_{a17} - s_{a15}w_{a17} - s_{e16}w_{a17} + w_{a17} - 2)}{b_{a17}s_{a15}s_{e16} - b_{a17}s_{a15} + s_{a15}s_{b14}w_{a16} + 1}, \]
\[ s_{a15} = -s_{b14}w_{a14}w_{a16} + 1, \]
\[ b_{a14} = -w_{a14} + 1, \]
\[ w_{a14} = \frac{b_{a13}s_{a15}s_{b14} - b_{a13}s_{a15} + 1}{b_{a13}s_{a15}s_{b14} - b_{a13}s_{a15} + s_{a15}s_{b14}w_{a13} - s_{b14}w_{a13} + 2}, \]
The corresponding dynamical System

\[ b_{a13} = -w_{a13} + 1, \]
\[ w_{a13} = \frac{b_{a12} b_{a14} s_{b13} - 1}{b_{a12} b_{a14} s_{b13} - s_{b13} w_{a12} w_{a14} + w_{a12} w_{a14} - 2}, \]
\[ b_{a12} = -w_{a12} + 1, \]
\[ w_{a12} = \frac{b_{a11} b_{a13} s_{b13} - 1}{b_{a11} b_{a13} s_{b13} - s_{b13} w_{a11} w_{a13} + w_{a11} w_{a13} - 2}, \]
\[ b_{a11} = -w_{a11} + 1, \]
\[ w_{a11} = \frac{b_{a10} b_{a12} s_{f10} - b_{a10} b_{a12} + 1}{b_{a10} b_{a12} s_{f10} + w_{a11} w_{a9} w_{b10} - 2}, \]
\[ b_{a10} = -w_{a10} + 1, \]
\[ w_{a10} = \frac{b_{a11} b_{a9} b_{b10} - 1}{b_{a11} b_{a9} b_{b10} + w_{a11} w_{a9} w_{b10} - 2}, \]
\[ b_{a9} = -w_{a9} + 1, \]
\[ w_{a9} = \frac{b_{a10} b_{a8} b_{b9} - 1}{b_{a10} b_{a8} b_{b9} + w_{a10} w_{a8} w_{b9} - 2}, \]
The corresponding dynamical System

\[ b_{a8} = - w_{a8} + 1, \]
\[ w_{a8} = \frac{b_{a7} b_{a9} b_{b8} - 1}{b_{a7} b_{a9} b_{b8} + w_{a7} w_{a9} w_{b8} - 2}, \]
\[ b_{a7} = - w_{a7} + 1, \]
\[ w_{a7} = \frac{b_{a6} b_{a8} b_{b7} - 1}{b_{a6} b_{a8} b_{b7} + w_{a6} w_{a8} w_{b7} - 2}, \]
\[ b_{a6} = - w_{a6} + 1, \]
\[ w_{a6} = \frac{b_{a5} b_{a7} b_{b6} - 1}{b_{a5} b_{a7} b_{b6} + w_{a5} w_{a7} w_{b6} - 2}, \]
\[ b_{a5} = - w_{a5} + 1, \]
\[ w_{a5} = \frac{b_{a4} b_{a6} b_{b5} - 1}{b_{a4} b_{a6} b_{b5} + w_{a4} w_{a6} w_{b5} - 2}, \]
\[ b_{a4} = - w_{a4} + 1, \]
\[ w_{a4} = \frac{b_{a3} b_{a5} s_{b4} - b_{a3} b_{a5} + 1}{b_{a3} b_{a5} s_{b4} - b_{a3} b_{a5} - s_{b4} w_{a3} w_{a5} + 2}. \]
The corresponding dynamical System

\begin{align*}
b_{a3} &= - w_{a3} + 1, \\
\frac{b_{a2} b_{a4} s_{b3} - 1}{b_{a2} b_{a4} s_{b3} - s_{b3} w_{a2} w_{a4} + w_{a2} w_{a4} - 2} &= w_{a3} \\
b_{a2} &= - w_{a2} + 1, \\
\frac{b_{a1} b_{a3} b_{b2} - 1}{b_{a1} b_{a3} b_{b2} + w_{a1} w_{a3} w_{b2} - 2} &= w_{a2} \\
b_{a1} &= - w_{a1} + 1, \\
\frac{b_{a2} b_{b1} - 1}{b_{a2} b_{b1} + w_{a2} w_{b1} - 2} &= w_{a1}
\end{align*}
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Statistics on Professional Games

A batch mode testing environment has been written to compare the 10.4 million moves of 50,000 professional GoGoD games each with best moves as proposed by the influence module (IM). This produces the following statistics:

- for each move number a diagram showing the rank distribution of the professional move according to the influence module
- a listing of board positions where the professional move ranked lowest (most urgent improvements for IM)
- time measurements
Figure: A statistics of the ranking of the next professional move after move 100 in 50000 professional games
To help improving the influence field to become useful as an evaluation function in a game playing program the following interfaces have been written:

- a runtime library libgotoolse.so for linux (and DLL for Windows if needed) including a program in Pascal and in C (by Sam Owre) to test the runtime library,
- a possibility to run autoplay games on Fuego between the influence module move selection and move selection by other Fuego players (by SO),
- a possibility to run Fuego with IM move selection on CGOS (Computer GO Server) against other programs (by SO).
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Lessons Learned

- Can the pure dynamical system approach recognize unconditional life?

  → Only if based on 1-point eyes, otherwise only approximately which is not good enough and unnecessarily expensive.

  → How quickly is the system converging? Very fast with only few iterations being necessary in incremental mode, takes <100 µsec for the whole 19×19 board on a 3 GHz PC.

  → How many (real) solutions with values in the interval 0...1 does the dynamical system have? In general only one with the only exception of two opposite colour chains attached to each other, both with only one liberty which is a very unstable situation in Go.
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Does the solution of the iteratively solved dynamical system depend on the used initial values?

This implies that one can not iterate: to compute influence, to use influence values to compute life/death values, to use them as improved initial data of influence ... because the influence computation is stable against changing initial values.

Should the influence/evaluation function depend on and make use of who moves next?

not in its first approximation, only in higher order approximations when interlinking it with treesearch for its improvement (see next project).
More Learned

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The End