The Caribou Mathematics Contest

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May 13, 2010

Outline

Much Variation in Extra Curricular Mathematics

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- ► On one hand we tried to provide challenges for exceptionally talented kids and on the other hand we were a meeting point to get help on math homework.
- ► Some material we taught originated from the Eastern European tradition other is home-grown.

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The Caribou contest is one possible approach.

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- ▶ school level: 3 hours for 4 questions
- ▶ regional and higher levels: 3 hours for 3 questions on each of 2 days
- ▶ free training camps for the top students on regional level: each school year 2-3 weekend camps and winter and summer 2-week camps with university mathematics students and doctors giving lectures about set-theory, number theory, geometry, linear algebra, analysis, logic, combinatorics, probability, ..., up to measure theory with a contest at the end of each camp.

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- to show interested children that they are not the only ones in the world who like math and math puzzles (even if they do not know other children in their class or among their friends who like math),

- to make math and math puzzling a topic to be talked about, to give kids interested in math the confidence to stand up for their interest, (currently anyone good in math is treated as an alien or 'rocket scientist', i.e. as someone outside the community by the media),

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- to be a mentor of mathematically interested children whose needs are not of central concern of the school system, not of the society and even not of their parents who may not be interested in math, or do not think it has any relevance compared to socially highly respected skills, for example, hockey,
- to make it worthwile for math clubs at schools to be opened, now having 3 math contests for each age group available per year.

Outline

Stages of a Problem I

We break up the creation of a new contest into a number of steps:

- an idea for a problem, a translation from a problem book,..
- a proper English version with grade and number of marks,
- an added French translation,
- an html encoding of both,
- a cut and paste insertion into the database including references between the test database and the question database,
- to some problems the adding of an illustration,
- a final check and if necessary back to step 2.

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After all steps are completed and the problems have been
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Anna blows once on a dandelion seed head. She blows a second time and 3 times as many seeds as the first time fly off. Toni blows once and 2 times as many seeds fly off as did with both of Anna's blows combined. There was total of 60 seeds. What is the difference between Anna's first blow and Toni's blow?

Outline

Flexibility

We try to make questions entertaining, to require little mathematical knowledge but imagination and thinking out of the box.

Example:

A blacksmith wants to connect 5 chains, each containing 3 links, into one long chain by opening several links and then closing them. The least number of links to open is:

a) 3   b) 4   c) 5   d) 6 .

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Partial Credit

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- ▶ student may be lucky in guessing
  $\rightarrow$ more options for harder questions
- ▶ no partial credit
  $\rightarrow$ have some wrong options that can be excluded easier than others.

Example for Partial Credit

The task is to find which digit each letter represents. Which of the following statements is true:

F=1   F=2   F=6   F=8   F=9 ?

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A  +   A  =   B
×      +      +
B  ×  AC  =  BD
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The simplest chain of conclusions is:

1. column → A=1
1. row   → B=2
2. column → C≠0, C≥3
2. column → E≥4
3. row   → F≥8, F even → option 4.

If one is able to draw the first conclusion then option 1 can be dropped and the chance to guess the right solution increases from 20 to 25%.

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If one is able to draw the first conclusion then option 1 can be dropped and the chance to guess the right solution increases from 20 to 25%.

From the second conclusion options 2 and 5 can be dropped and the chance to guess the right solution increases to 50%.

Preventing Checking Option

Another problem with multiple choice questions occurs when it is much easier to simply check the different options than to solve the problem.

The possible solution to this problem is to give only partial information in the options which is not enough to simply check this option.

Example for Preventing Checking Option

There are 25 books on two shelves. On one shelf there are 3 books more than on the other shelf. How many books are on each shelf? Which of the following statements is correct?

a) One shelf has more than 15 books.
b) The number of books on one shelf is divisible by 5.
c) One shelf has less than 10 books.
d) One shelf has twice as many books as the other.
e) The number of books on one shelf is a multiple of 7.

Logic Problems

Part of mathematics is to be able to be precise if necessary. This has nothing to do with difficulty or complexity, just with practise and being required from time to time to formulate logical statements precisely and to recognize logically wrong conclusions.

The following question type was motivated by serious weaknesses of our mathematics students when more than computation is required.

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   c) *If the Pittsburgh Penguins do not win the game, then Sidney Crosby does not score an overtime goal.*

   d) *If the Pittsburgh Penguins do not win the game, then Sidney Crosby scores an overtime goal.*

   e) *None of the other statements is always true.*

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Between Jan 2008 and 22 April 2010 a practise test has been taken 14301 times. Thus, in selecting questions we believe we have some influence on what children learn.

Outline

Who are the Winners?

Our first contest had a respectable number of 598 participants. We had organized many different prizes with a total value of $1000: tickets for an art gallery, for Merrittville car racing, for minigolf, for an OHL ice hockey game, swim passes, books, a signed hockey stick of the local OHL team (Ice Dogs), cheques, gift cards of super stores, toys, ...

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The most wanted prize by far was the signed hockey stick.
$\rightarrow$ The stereotype of children good in math being nerds is wrong!

Participating Schools

- ▶ Relatively high percentage of private schools which are in general better funded than public schools for which our free contest should be more interesting.

- ▶ We are not comparing schools which would not be fair because some are schools of gifted children and others not.

- ▶ But when checking the lists of top contestants one school is very dominant in all contests: Terry Fox School Toronto. In the last contest of April 21, 2010 they got the top 3 places and 10 out of the top 13.

Outline

Our Plans

- Planned extensions for next school year 2010/2011: extra grade 7/8, contest open Canada wide
- Questions may have a varying number of options to allow fewer questions of which some are relatively difficult and with more options.
- For grades 5/6 and 7/8 we plan to have a logic question and a calcrostic question in each contest.
- We will most likely have to stop giving out prizes.
- Funding for continuing the contest in the coming year has recently been secured through support by the Fields Institute Toronto.

Questions and Hints to Teachers

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- ▶ Other ideas for reaching more children?
- ▶ How important are prizes?

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