Calcrostics - a marathon test for the algebraic solution of polynomial systems

Thomas Wolf
Department of Mathematics,
Brock University

St.Catharines, Ontario, Canada
email: twolf@brocku.ca

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Outline

A Need for Mathematical Puzzles

The online Caribou mathematics contest
`www.brocku.ca/caribou`
is held Ontario wide 6 times in the school year 2009/10.

It will be held Canada wide 9 times in the coming school year 2010/11.

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For each contest 24 questions (and their French translation) are needed.

In addition the contest home web page needed a 'Problem of the Day'.

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Such puzzles are harder to create - a good task for computers.

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A known Type of Puzzle

Puzzles, like

$$\begin{array}{rcrcr} \text{EDKH} & \div & \text{KF} & = & \text{AA} \\ - & & + & & + \\ \text{EDB} & \times & \text{J} & = & \text{EHCG} \\ = & & = & & = \\ \text{EEJD} & - & \text{DK} & = & \text{EEAE} \end{array}$$

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This one has the solution

$$\begin{array}{rcrcr} 1320 & \div & 24 & = & 55 \\ - & & + & & + \\ 137 & \times & 8 & = & 1096 \\ = & & = & & = \\ 1183 & - & 32 & = & 1151 \end{array}$$

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Brute force guessing: There are $10! = 3,628,800$ possible permutations of (0,1,2,..), i.e. maps (A,B,C,..) $\rightarrow$ (0,1,2,..) but an educated guessing organized in a genetic learning program needs on average just 1000 guesses.

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- we can require extra identities on the diagonals.

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We call these problems **calcrostic** because they are showing **cal**culations in a form similar to a**crostic** word puzzles.

Example:

AB	$\times$	C	=	DEA
+	$\times$	$\div$	$\div$	$-$
AB	$\times$	B	=	EF
=	=	=	=	=
BC	$\times$	A	=	EF

Table: A puzzle (P1).

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24	$\times$	8	=	192
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Apart from re-labeling digits with other letters/symbols there are the following equivalence transformations.

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Equivalent Calcrostics

For example, starting with

24	×	8	=	192
+	×	÷	÷	−
24	×	4	=	96
=	=	=	=	=
48	×	2	=	96

Table: Solution (S1).

we can swap the first and third row, invert the 3 column operations, swap and invert the diagonal operations and get

48	×	2	=	96
−	×	×	÷	+
24	×	4	=	96
=	=	=	=	=
24	×	8	=	192

Table: A new solution (S2).

AB	×	C	=	DE
−	×	×	÷	+
CA	×	A	=	DE
=	=	=	=	=
CA	×	B	=	FDC

Table: Its encoding (P2).

Equivalent Calcrostics

By swapping the 1st and 3rd column we obtain from S2:

96	$\div$	2	=	48
+	$\div$	$\times$	$\times$	−
96	$\div$	4	=	24
=	=	=	=	=
192	$\div$	8	=	24

Table: A new solution (S3).

AB	$\div$	C	=	DE
+	$\div$	$\times$	$\times$	−
AB	$\div$	D	=	CD
=	=	=	=	=
FAC	$\div$	E	=	CD

Table: Its encoding (P3).

Swapping again 1st and 3rd row in S3 gives us S4:

192	$\div$	8	=	24
−	$\div$	$\div$	$\times$	+
96	$\div$	4	=	24
=	=	=	=	=
96	$\div$	2	=	48

Table: A new solution (S4).

ABC	$\div$	D	=	CE
−	$\div$	$\div$	$\times$	+
BF	$\div$	E	=	CE
=	=	=	=	=
BF	$\div$	C	=	ED

Table: Its encoding (P4).

S4 could also be generated from S1 by swapping 1st and 3rd column and inverting some of the operations appropriately.

Equivalent Calcrostics

Starting again from solution S1, by mirroring on the main diagonal gives S5:

24	+	24	=	48
×	×	×	×	×
8	÷	4	=	2
=	=	=	=	=
192	−	96	=	96

Table: A new solution (S5).

AB	+	AB	=	BC
×	×	×	×	×
C	÷	B	=	A
=	=	=	=	=
DEA	−	EF	=	EF

Table: Its encoding (P5).

Swapping rows 1 and 3 in S5 gives S6:

192	−	96	=	96
÷	÷	÷	÷	÷
8	÷	4	=	2
=	=	=	=	=
24	+	24	=	48

Table: A new solution (S6).

ABC	−	BD	=	BD
÷	÷	÷	÷	÷
E	÷	F	=	C
=	=	=	=	=
CF	+	CF	=	FE

Table: Its encoding (P6).

Equivalent Calcrostics

Swapping columns 1 and 3 in S6 gives S7:

96	+	96	=	192
÷	÷	÷	÷	÷
2	×	4	=	8
=	=	=	=	=
48	−	24	=	24

Table: A new solution (S7).

AB	+	AB	=	CAD
÷	÷	÷	÷	÷
D	×	E	=	F
=	=	=	=	=
EF	−	DE	=	DE

Table: Its encoding (P7).

Finally, swapping rows 1 and 3 in S7 gives S8:

48	−	24	=	24
×	×	×	×	×
2	×	4	=	8
=	=	=	=	=
96	+	96	=	192

Table: A new solution (S8).

AB	−	CA	=	CA
×	×	×	×	×
C	×	A	=	B
=	=	=	=	=
DE	+	DE	=	FDC

Table: Its encoding (P8).

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  Yes, all solutions satisfy the same system of 8 equations up to permutation of letters and a rewriting of individual equations, e.g. $a + b = c \leftrightarrow c - b = a$. None of the two operations changes the number of solutions of the system.
- If one version involves all 4 operations (like P1), then an equivalent version may involve fewer operations though (e.g. P8 has no $\div$).

Outline

A Total Ordering of Calcrositics

From now on the term 'calcrostic' shall denote the group of 8 equivalent solutions and their encodings.

The question of membership of a specific calcrostic in a given large set is easy if one has a total ordering of them.

This is equivalent to having an algorithm to generate all calcrostics, one after another.

The following is a natural ordering.

Standard Form

For the following discussion we write the numbers in the puzzle in the form of a 3x3 matrix

$$\begin{matrix} a & b & c \\ d & f & g \\ h & k & m \end{matrix}$$

where lower case $a, b, ..$ denote whole numbers, not digits.
At first we use horizontal and vertical reflection to determine a rotation which makes the upper left corner minimal compared to all other 3 corners in the following sense. For example, the comparison between the two top corners would be:

$$\begin{aligned} &(a+b+c) < (b+c+g) \quad \text{or} \\ &((a+b+c) = (b+c+g) \quad \text{and} \quad (\ (a<c) \ \text{ or} \\ &\qquad\qquad ((a=c) \ \text{ and } \ (\min(b,d) < \min(b,g))))). \end{aligned}$$

Then we use the diagonal mirroring to have

$$d > b \text{ or } (d = b \text{ and } (h > c \text{ or } (h = c \text{ and } k \geq g))).$$

Generation of individual Calcrostics

A computer program searching for calcrostics involves many nested loops. From the 9 numbers

$$\begin{array}{ccc} a & b & c \\ d & f & g \\ h & k & m \end{array}$$

one needs only one loop variable (the sum $s := a + b + d + f$) to go to infinity which ensures for all 9 numbers to be of similar magnitude and avoids to generate, for example, infinitely many puzzles with $a = 1$.

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The nested loops are: $\forall s = 4..\infty,\ \ \forall f = 1..(s-3),$ $\forall b = 1..(s-f-2),\ \ \forall d = 1..(s-f-b-1),\ \ \forall \text{operators}, ...$

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In this way all calcrostics in standard form with $s \leq 1000$ were generated including all their equivalent forms.

Generation of Families of Calcrostics

A different approach to generating calcrostics:

- fix operators first,
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For example,

$$\begin{array}{ccccc} a & + & b & = & c \\ \times & - & + & + & - \\ d & + & f & = & g \\ = & = & = & = & = \\ h & \div & k & = & m \end{array}$$

results in the (non-linear) system of equations:

$$\begin{array}{ccccccc} 0 & = & a+b-c & = & a\times d-h & = & a-f-m \\ 0 & = & d+f-g & = & b+f-k & = & c+f-h \\ 0 & = & h-k\times m & = & c-g-m & & \end{array}$$

Generation of Families of Calcrostics

It's general solution has one free parameter:

$$
\begin{array}{ccccc}
a & + & 2 & = & (a+2) \\
\times & - & + & + & - \\
2 & + & (a-2) & = & a \\
= & = & = & = & = \\
(2\times a) & \div & a & = & 2
\end{array}
$$

Comments:

- A general solution may have free parameters providing a whole family of calcrostics.
- By starting with an operator setting of a calcrostic that has been found numerically, one has a guarantee that the resulting polynomial system has a solution and that the general solution contains special solutions involving only positive integers.

Two free Parameters

This calcrostic even contains two free parameters:

$$
\begin{array}{ccccc}
bh^2 & - & bh & = & bh(h-1) \\
- & \times & \times & \div & + \\
h(bh-1) & \times & b(h-1) & = & bh(h-1)(bh-1) \\
= & = & = & = & = \\
h & \times & b^2h(h-1) & = & b^2h^2(h-1)
\end{array}
$$

In addition to an arbitrary number of puzzles of the earlier type, it provides a new type of question:

What are the numbers a, b, ... (NOT digits) that satisfy the following puzzle?

$$
\begin{array}{ccccc}
\mathrm{a} & - & 28 & = & \mathrm{c} \\
- & \times & \times & \div & + \\
\mathrm{d} & \times & \mathrm{f} & = & \mathrm{g} \\
= & = & = & = & = \\
7 & \times & \mathrm{k} & = & \mathrm{m}
\end{array}
$$

One could of course simplify the question and provide a few of the missing numbers.

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Beautification

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This is done by replacing all $=$ signs by $-$ signs, so

$$
\begin{array}{ccccc}
24 & \times & 8 & = & 192 \\
+ & \times & \div & \div & - \\
24 & \times & 4 & = & 96 \\
= & = & = & = & = \\
48 & \times & 2 & = & 96
\end{array}
\quad \longrightarrow \quad
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$$

with the requirement that each line being evaluated gives zero.

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By changing all $=$ to $-$ the symmetry is still broken by the requirement to evaluate lines from top to bottom and from left to right (but this is less obvious than the $=$ signs. :-)).

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$\rightarrow$ Could additional requirements be formulated?

Are there calcrostics where the value of EACH line is zero: horizontal, vertical, the main diagonal, the other diagonal AND ALL SHORTER PARALLEL DIAGONALS?

Existence Problem

Does there exist a setting of operators ($\circ$ in the diagram) which involves all 4 operators $+$, $-$, $\times$, $\div$ and allows numbers $a, b, ..., m$ to exist that give all lines, including off diagonal lines, the value zero?

a	$\circ$	b	$\circ$	c
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
d	$\circ$	f	$\circ$	g
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
h	$\circ$	k	$\circ$	m

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To have solutions involving only positive integer values, each line must include at least one minus sign. This narrows the search to problems of the form

$$\begin{array}{ccccc} a & \circ & b & \circ & c \\ \circ & - & \circ & - & \circ \\ b & \circ & f & \circ & b \\ \circ & - & \circ & - & \circ \\ h & \circ & b & \circ & m \end{array}$$

A new type of Calcrostic

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Table: An operator setting.

12	–	2	–	10
–	–	–	–	–
2	–	4	÷	2
×	–	+	–	–
6	+	2	–	8

Table: The unique solution.

The formulation and solution of over 13,000 polynomial system on a PC within 2 weeks represents a comprehensive test suite which even exposed one bug in the package CRACK that had not occured before.

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- more solutions for higher m, n

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- but polynomial system is harder to solve because: more unknowns and more equations,

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Another generalization:

- arbitrary sizes $m \times n, \quad m, n \geq 3$ possible
- contains mn numbers and $(2n-1)(2m-1) - mn$ operators and 'only' $3(m+n-2)$ conditions
- more solutions for higher m, n
- but polynomial system is harder to solve because: more unknowns and more equations, systems have more solutions.

Outline

References

[1] The Caribou Mathematics Contest
http://www.brocku.ca/caribou

[2] T. Wolf, A study of Genetic Algorithms solving a Combinatorial Puzzle. preprint (1997), 21 pages.
http://lie.math.brocku.ca/twolf/papers/puzzle.ps

Thank you.